New Directions in Channelized Receivers and Transmitters

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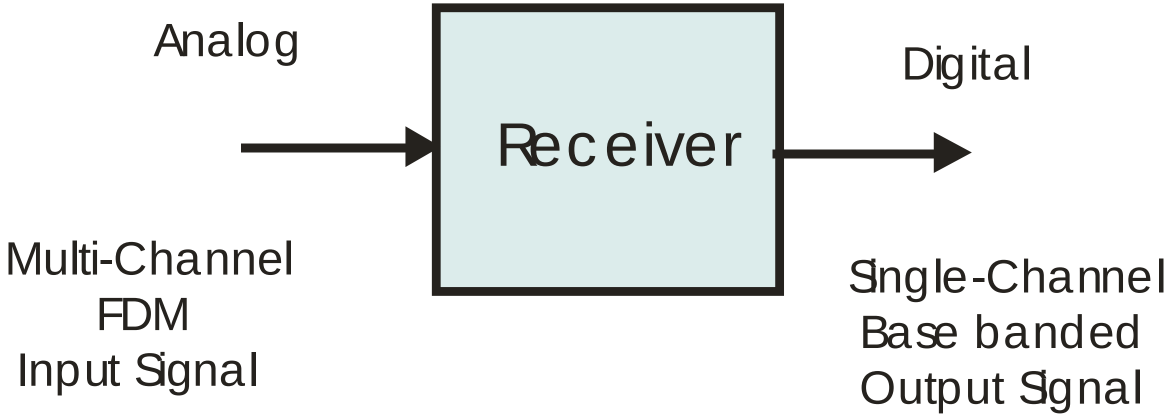
SAN DIEGO STATE UNIVERSITY
College of Engineering

SDR'11 - WInnComm

Motivation For Using Multirate Filters

Processing Task: Obtain Digital Samples of Complex Envelope Residing at Frequency f_C



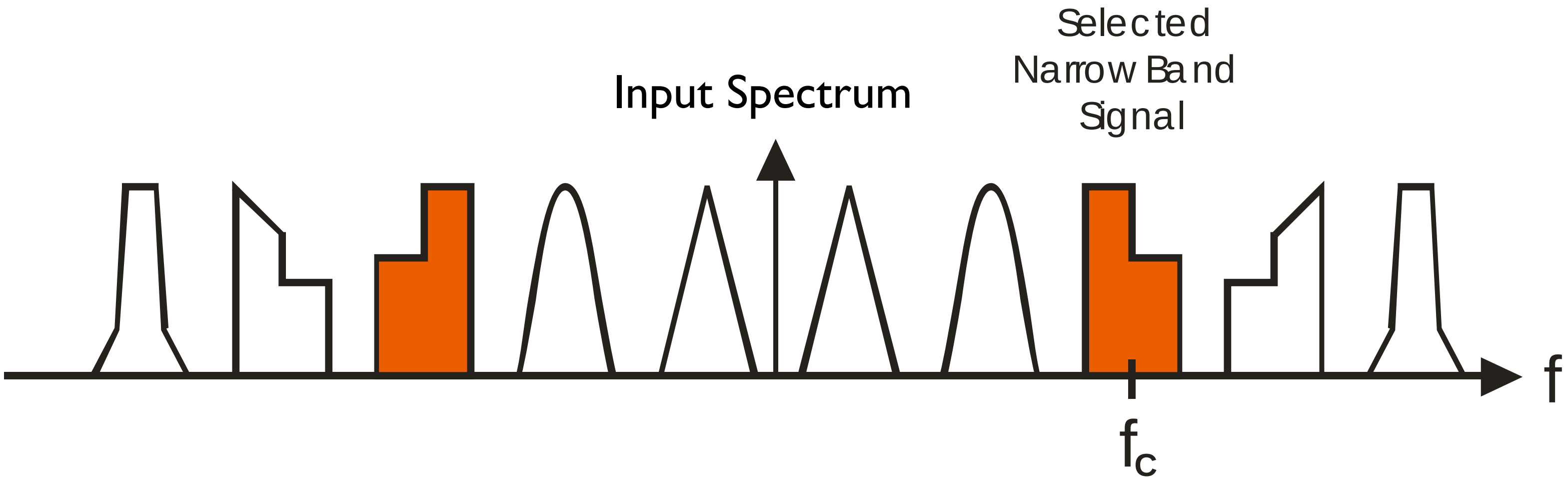
Analog

Receiver

Digital

Multi-Channel
FDM
Input Signal

Single-Channel
Base banded
Output Signal



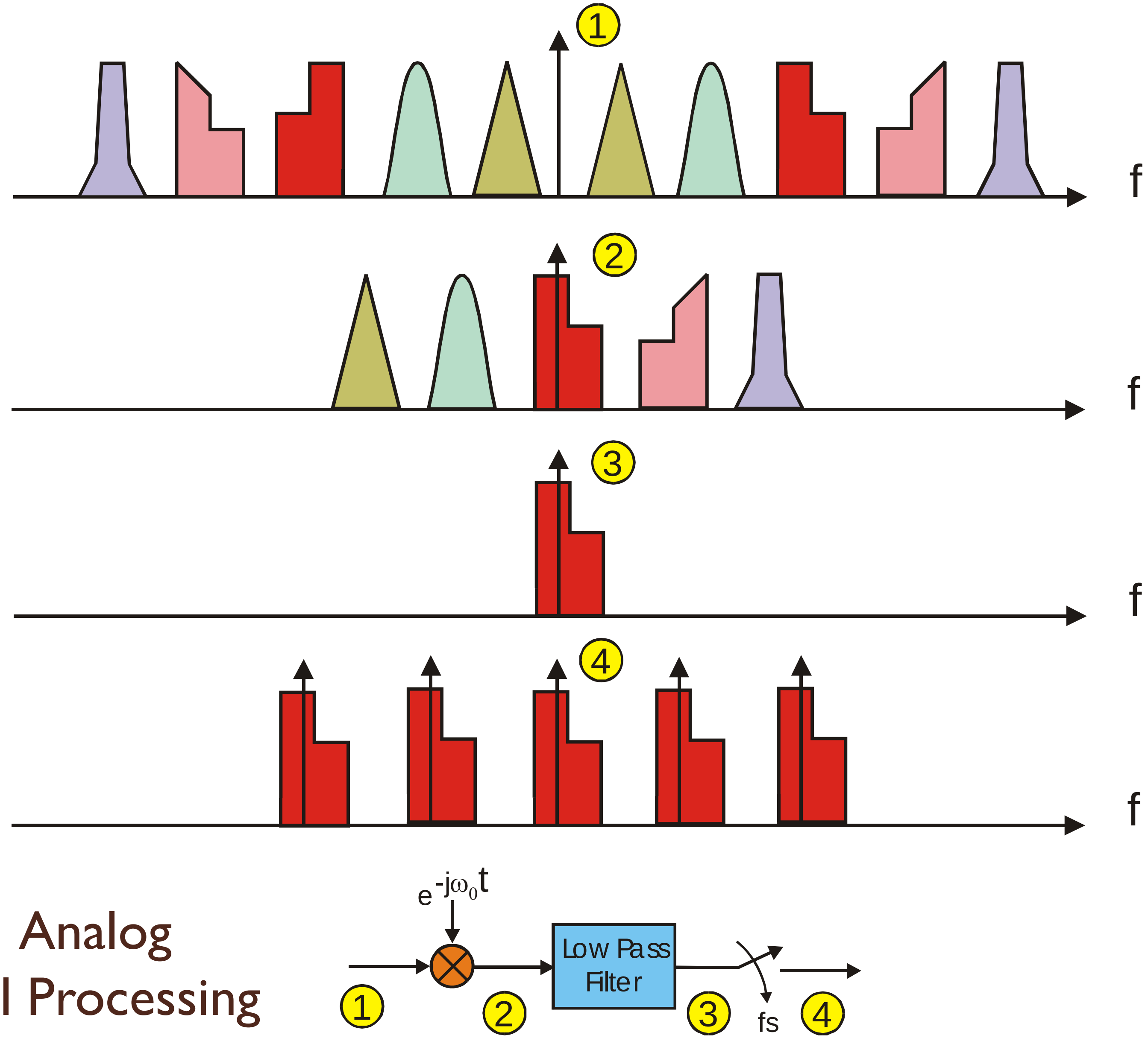
Selected
Narrow Band
Signal

Input Spectrum

See!

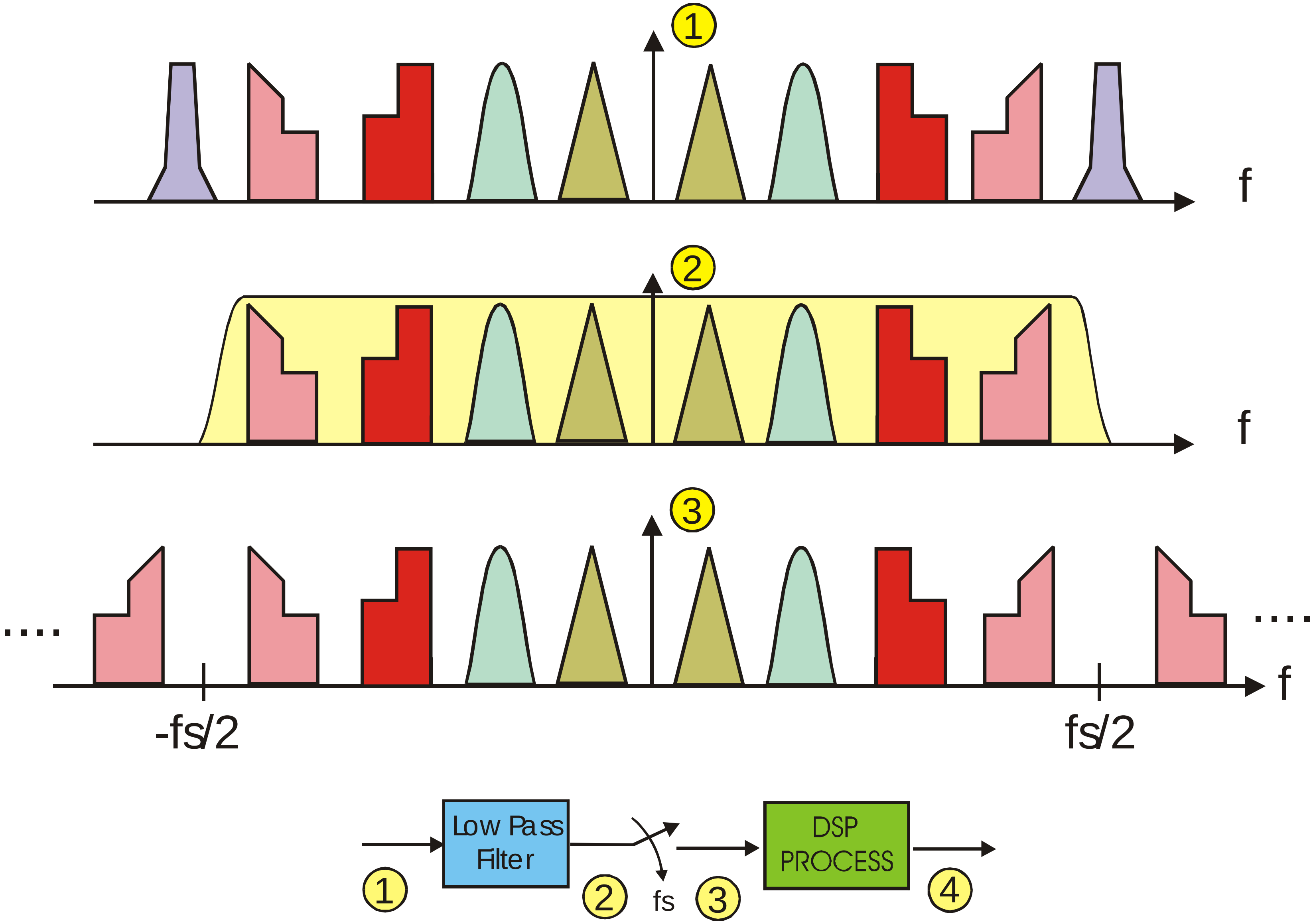
Washington
Oregon
Idaho
Montana
North Dakota
Minnesota
Maine
Vt.
N.H.
Mass.
Conn.
R.I.
New York
Wisconsin
Michigan
South Dakota
Wyoming
Nevada
Utah
California
Colorado
Nebraska
Iowa
Illinois
Indiana
Ohio
Pennsylvania
New Jersey
Md.
Delaware
D.C.
West Virginia
Virginia
Kansas
Missouri
Kentucky
North Carolina
Tennesee
Arizona
New Mexico
Oklahoma
Arkansas
South Carolina
Miss
Alabama
Georgia
Texas
Louisiana
Florida
Alaska
Hawaii

First Generation DSP Receiver

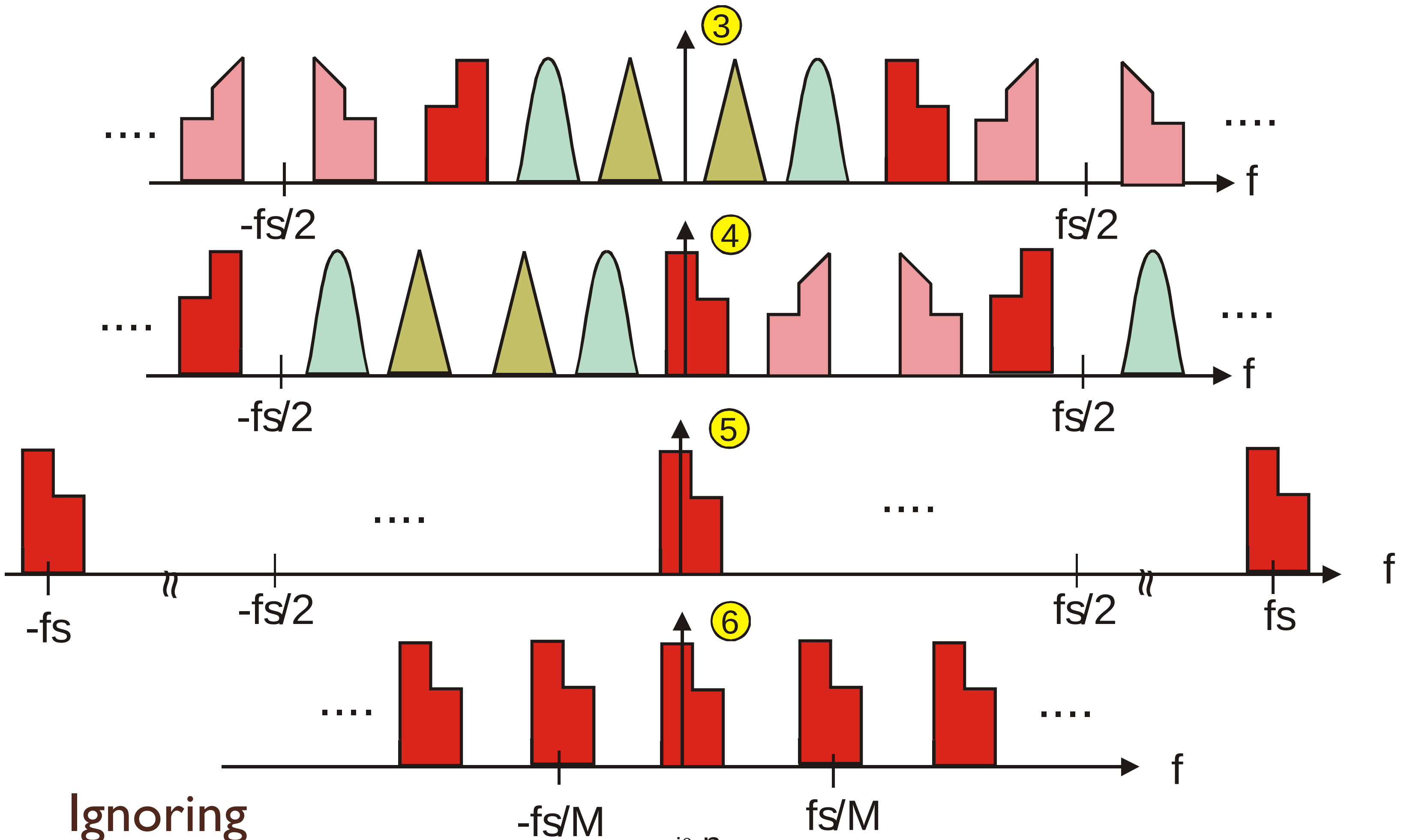


Analog
Signal Processing

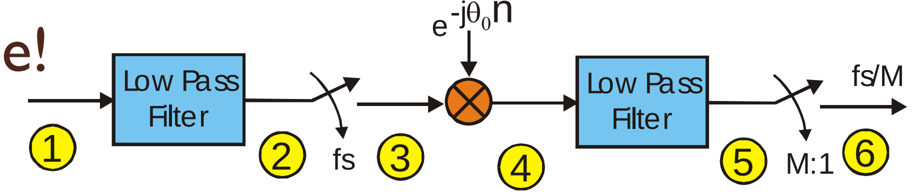
Signal Conditioning for DSP Receiver



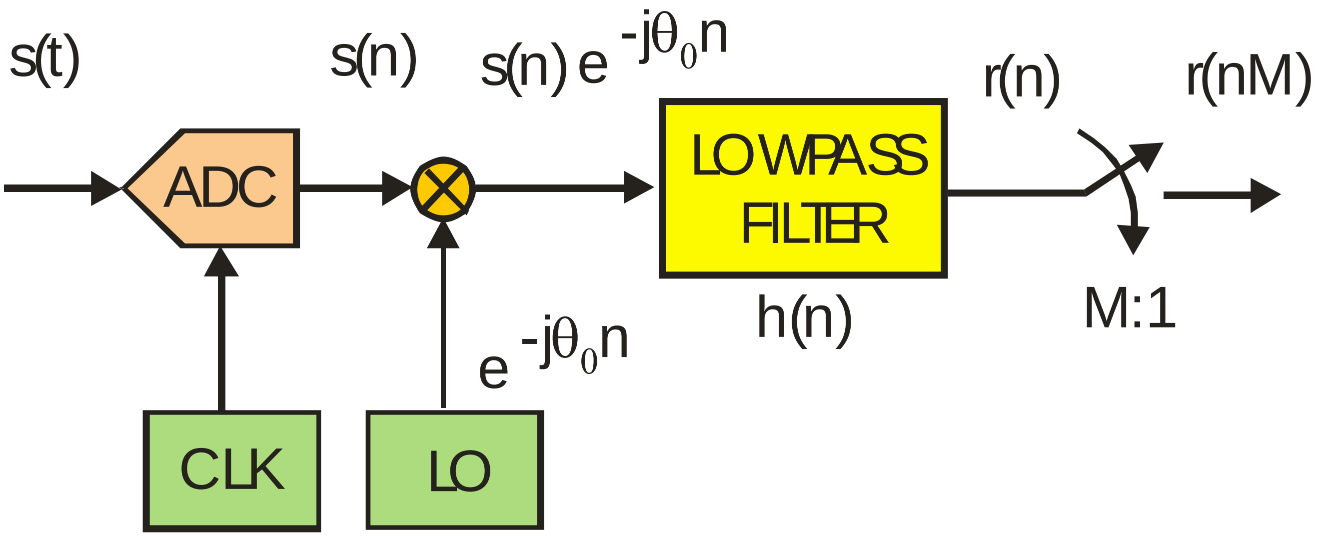
Replicate Analog Processing in DSP



Ignoring Good Advice!

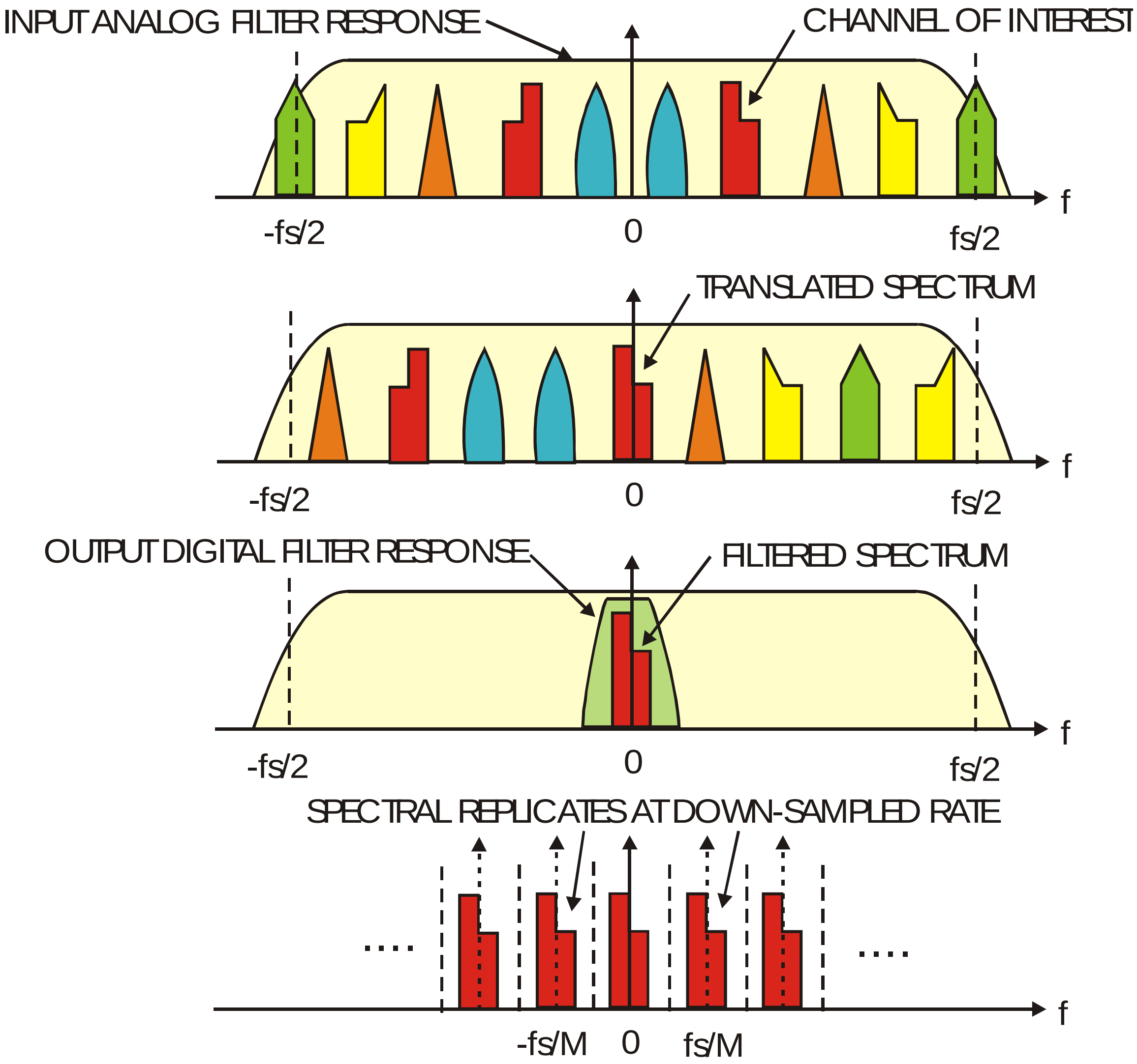


- Fundamental Operations
  - Select Frequency
  - Limit Bandwidth
  - Select Sample Rate

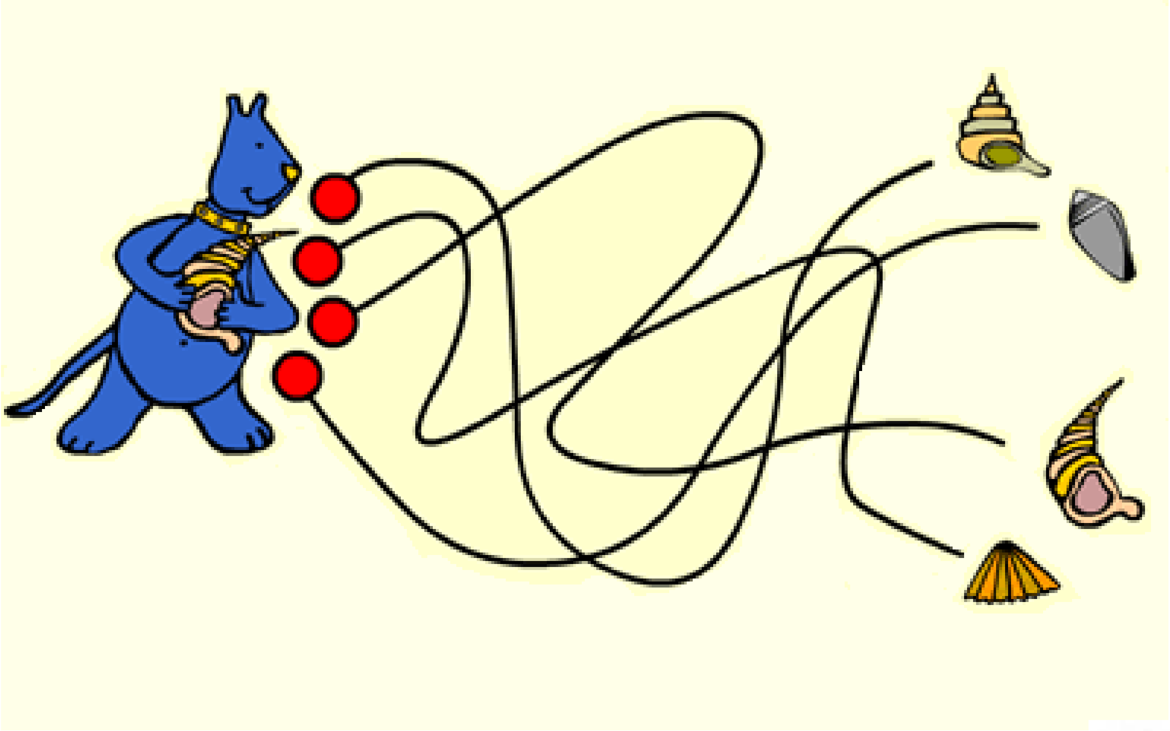


Digital Down Converter (DDC)

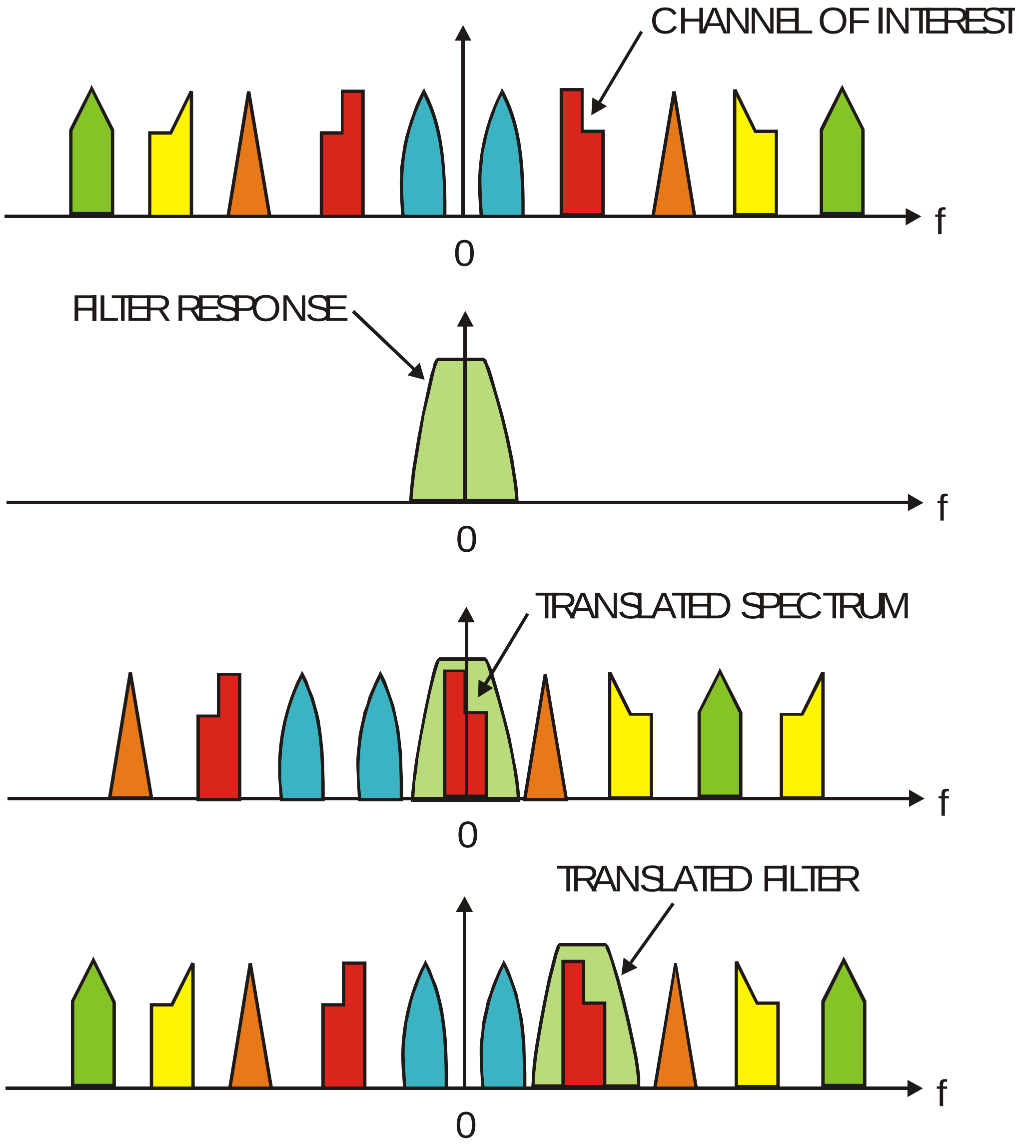
Spectral Description of Fundamental Operations



A Shell Game:
Rearrange the Players!
Keep Your Eye on the Pea!



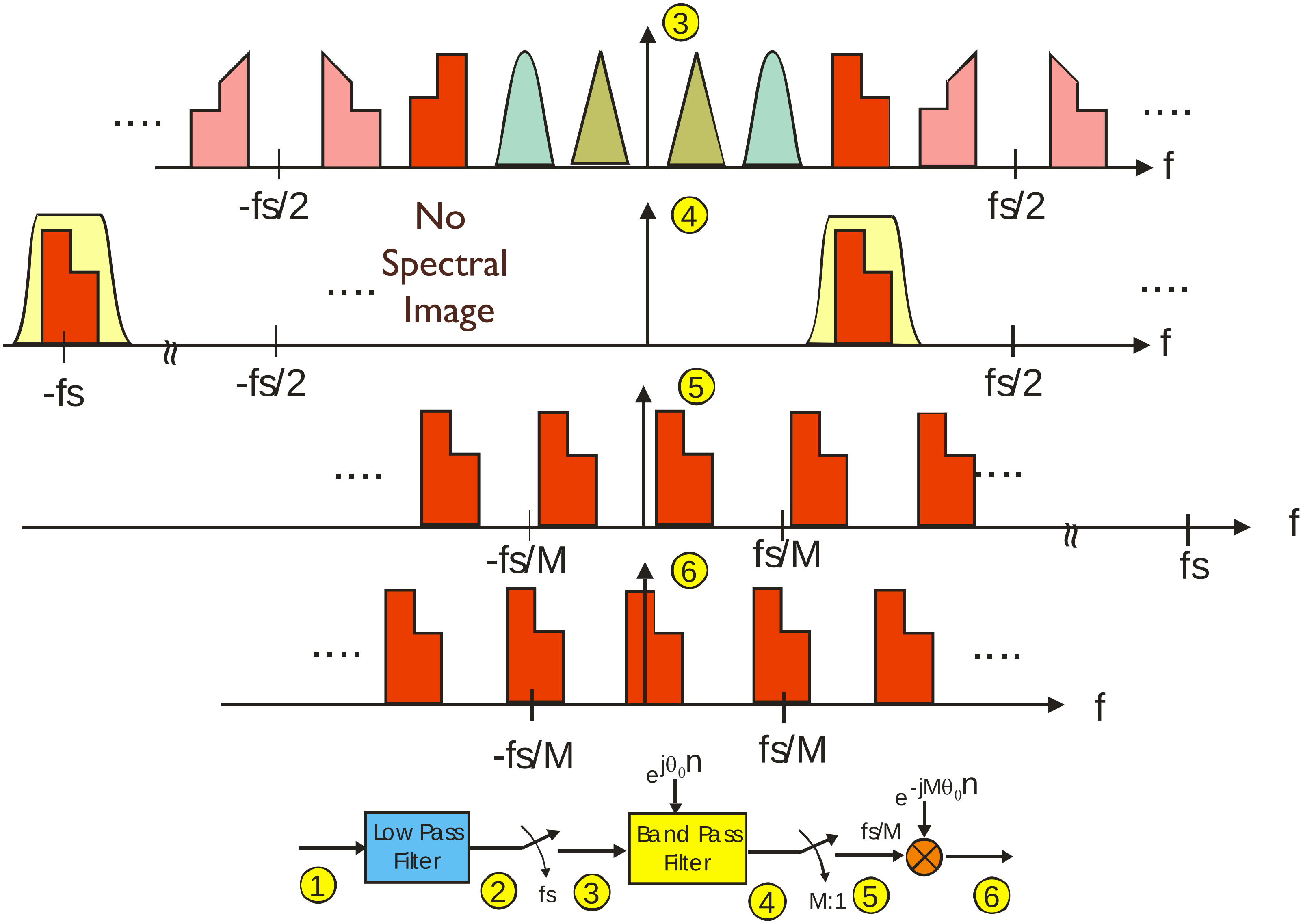
Signal and Filter are at Different Frequencies
Which One to Move?



Second
Option

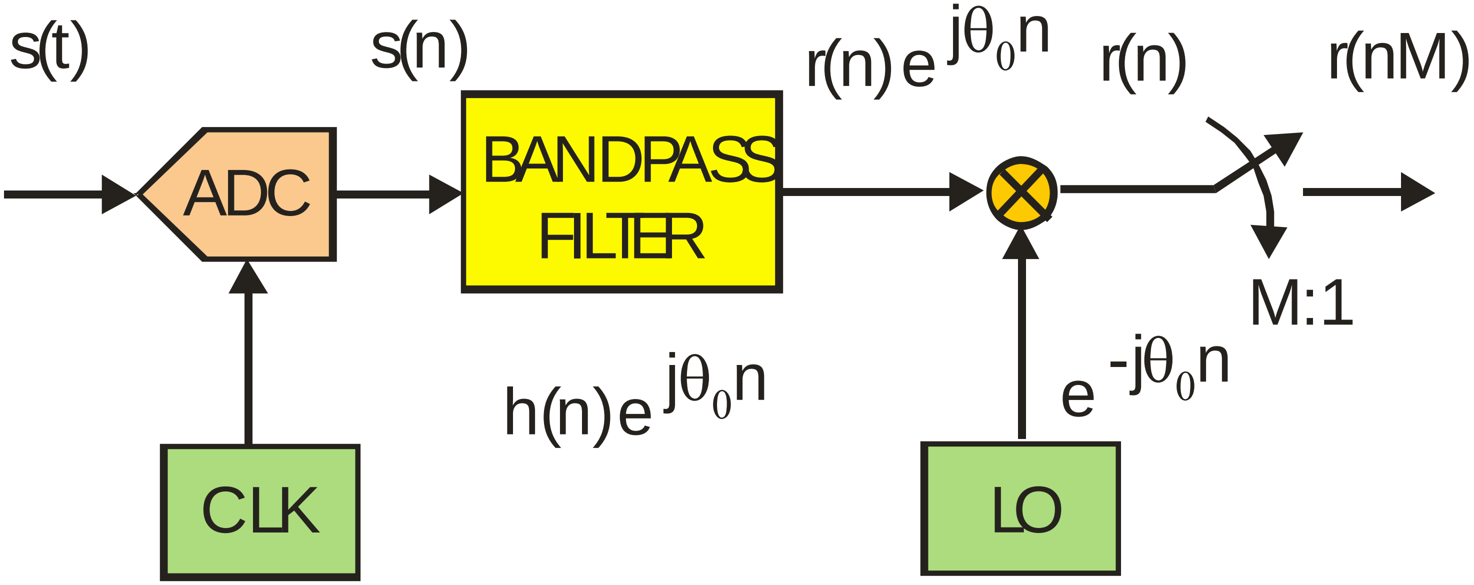
First
Option

Down Sample Complex Digital IF



Fundamental Operation with Rearrange Operators

Up-Convert Filter, Filter Signal at IF, Down Convert Output of Filter



Equivalency Theorem

Down-Convert Signal at
Input to Low-Pass Filter

$$r(n) = s(n)e^{-j\theta_0 n} * h(k)$$

$$= \sum_k s(n-k)e^{-j\theta_0 (n-k)} h(k)$$

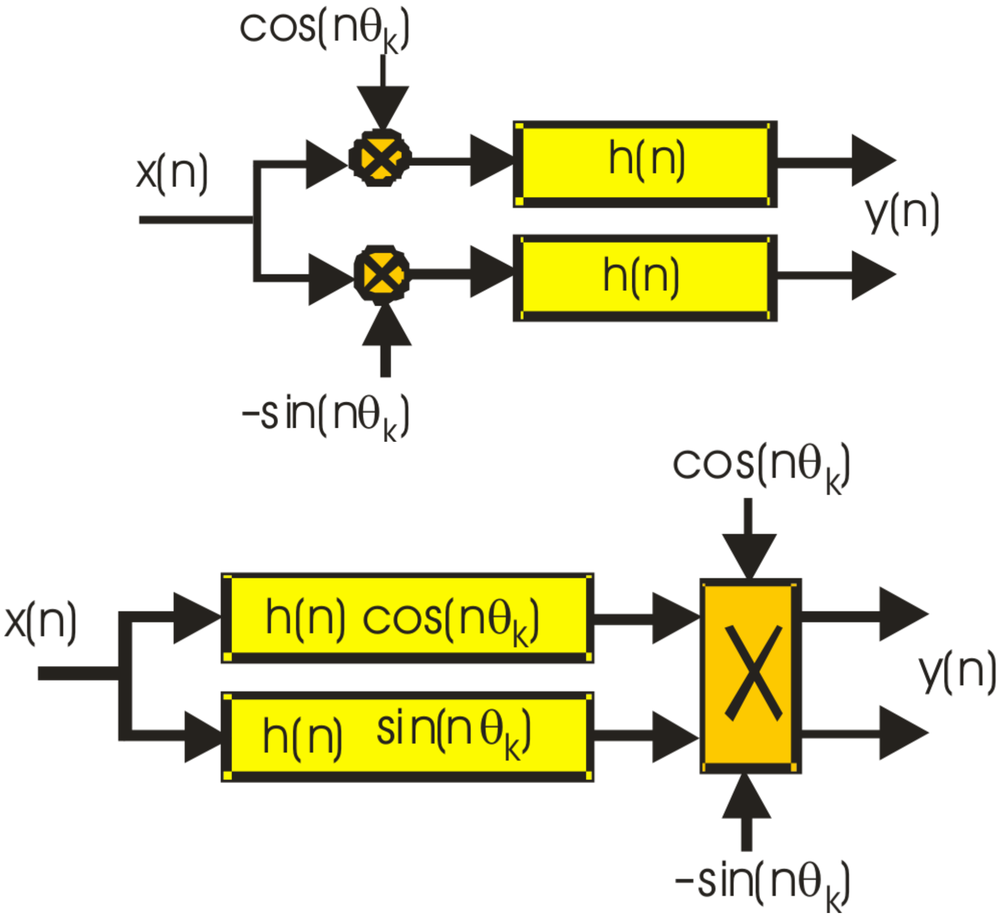
$$= e^{-j\theta_0 n} \sum_k s(n-k)h(k)e^{j\theta_0 k}$$

$$= e^{-j\theta_0 n} \{s(n) * h(n)e^{j\theta_0 n}\}$$

Down-Convert Signal at
Output of Band-Pass Filter

Up-Convert Low Pass Filter
To Become Complex
Band-Pass Filter

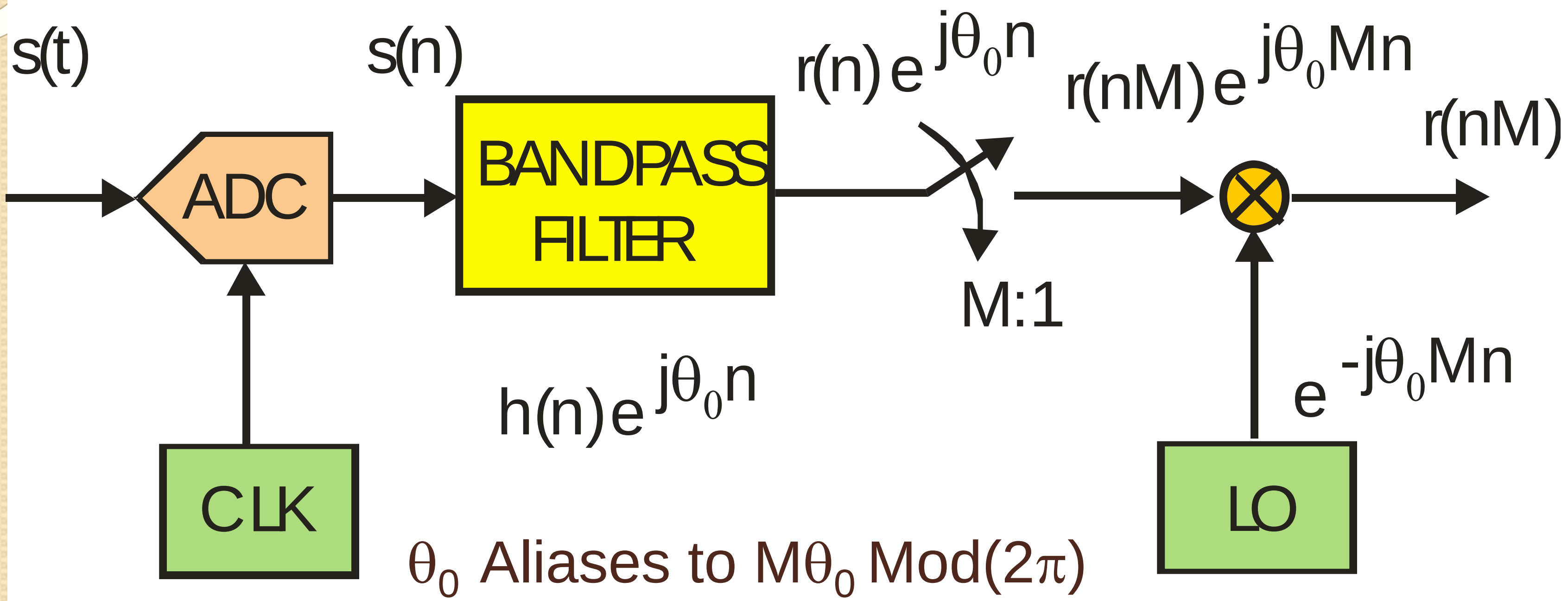
Signal Flow Description of Equivalency Theorem



Not Finished: Moving Down Converter from Input to Output

Replaces 2-Multipliers (Complex Scalar) with 4-Multipliers (Complex Product)

Interchange Down Converter and Resampler



θ_0 Aliases to $M\theta_0$ Mod(2π)

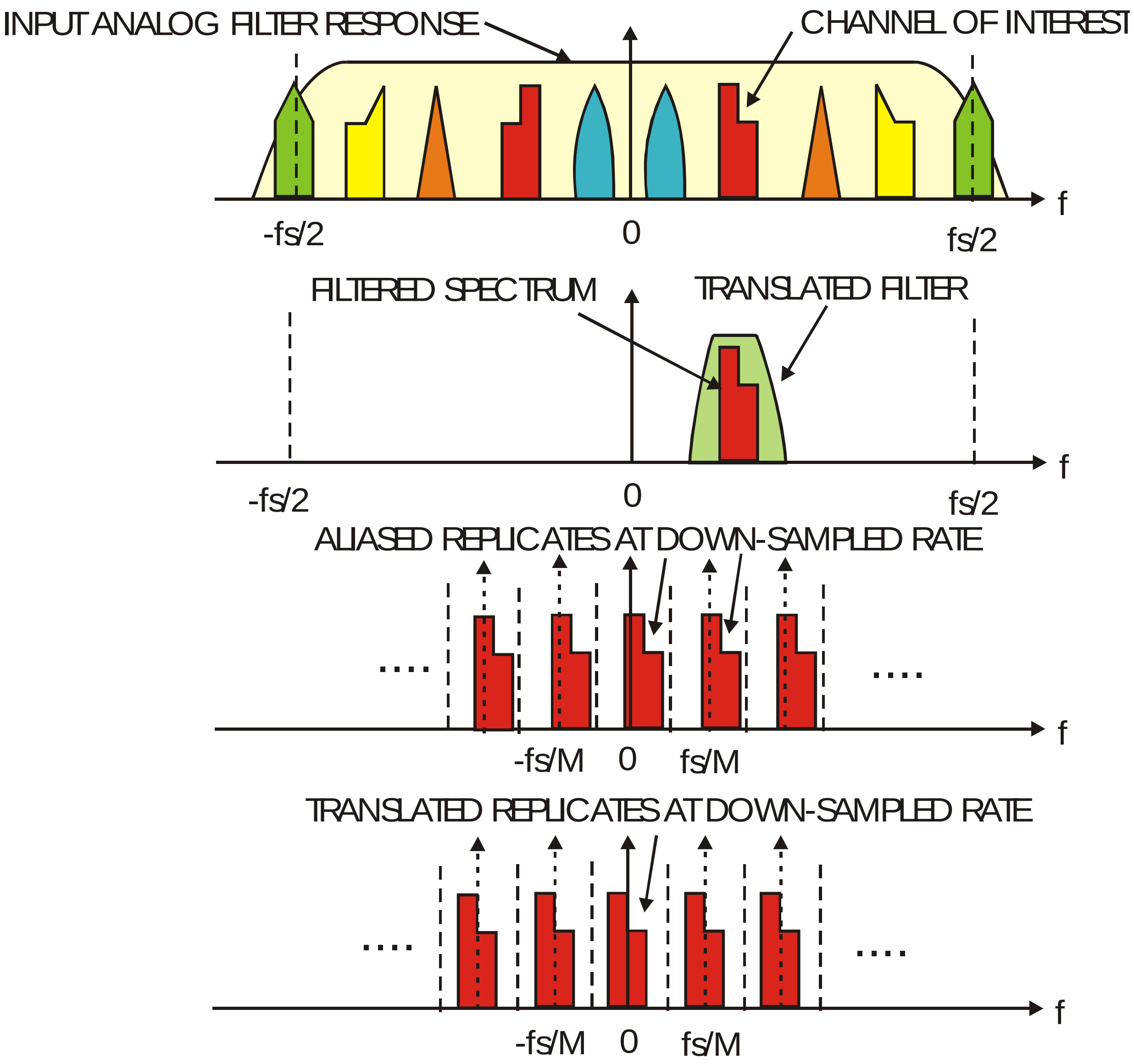
θ_0 Aliases to 0 if $M\theta_0 = k\cdot 2\pi$

Only Down Convert the Samples we Intend to Keep!

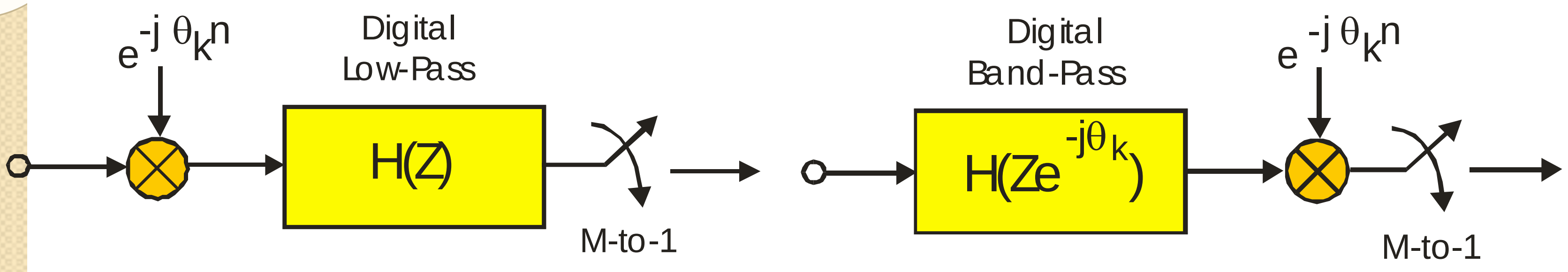
Let the Resampler Alias the Center Frequency to Baseband

SPECTRAL DESCRIPTION

REORDERED FUNDAMENTAL OPERATION



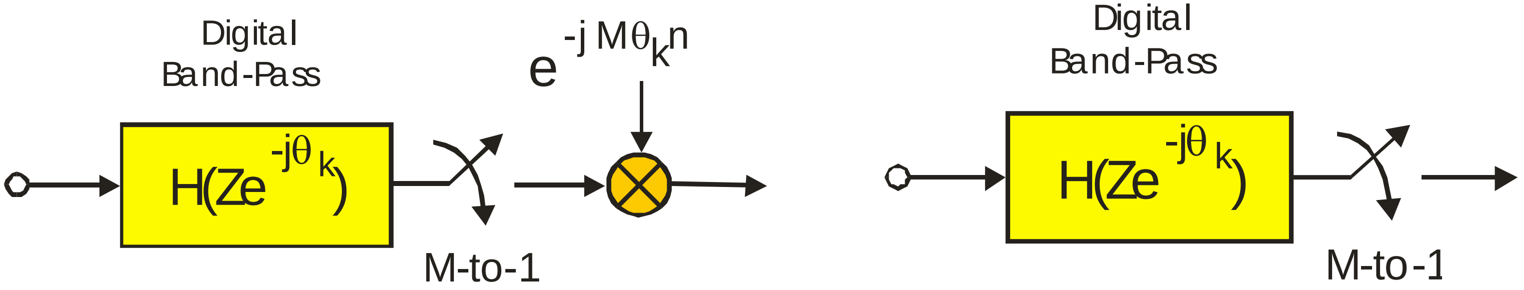
Successive Transformations Turn Sampled Data Version of Edwin Armstrong's Heterodyne Receiver to Tuned Radio Frequency (TRF) Receiver and then to Aliased TRF Receiver.



Armstrong

Nyquist

Equivalency Theorem

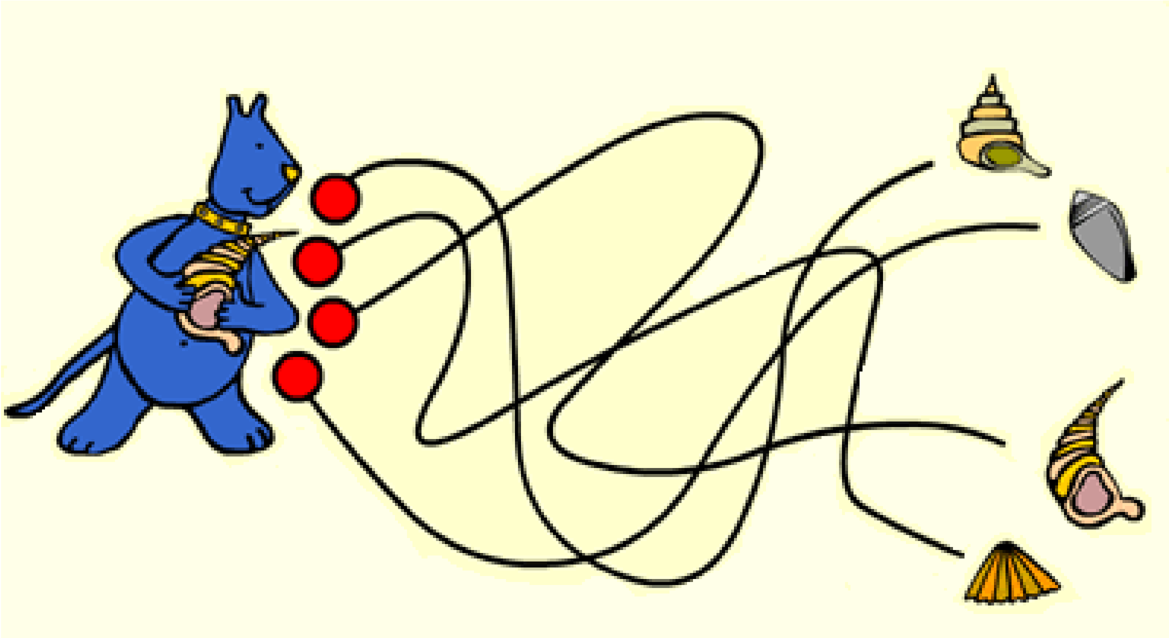


$$M \cdot \theta_k = k \cdot 2\pi$$

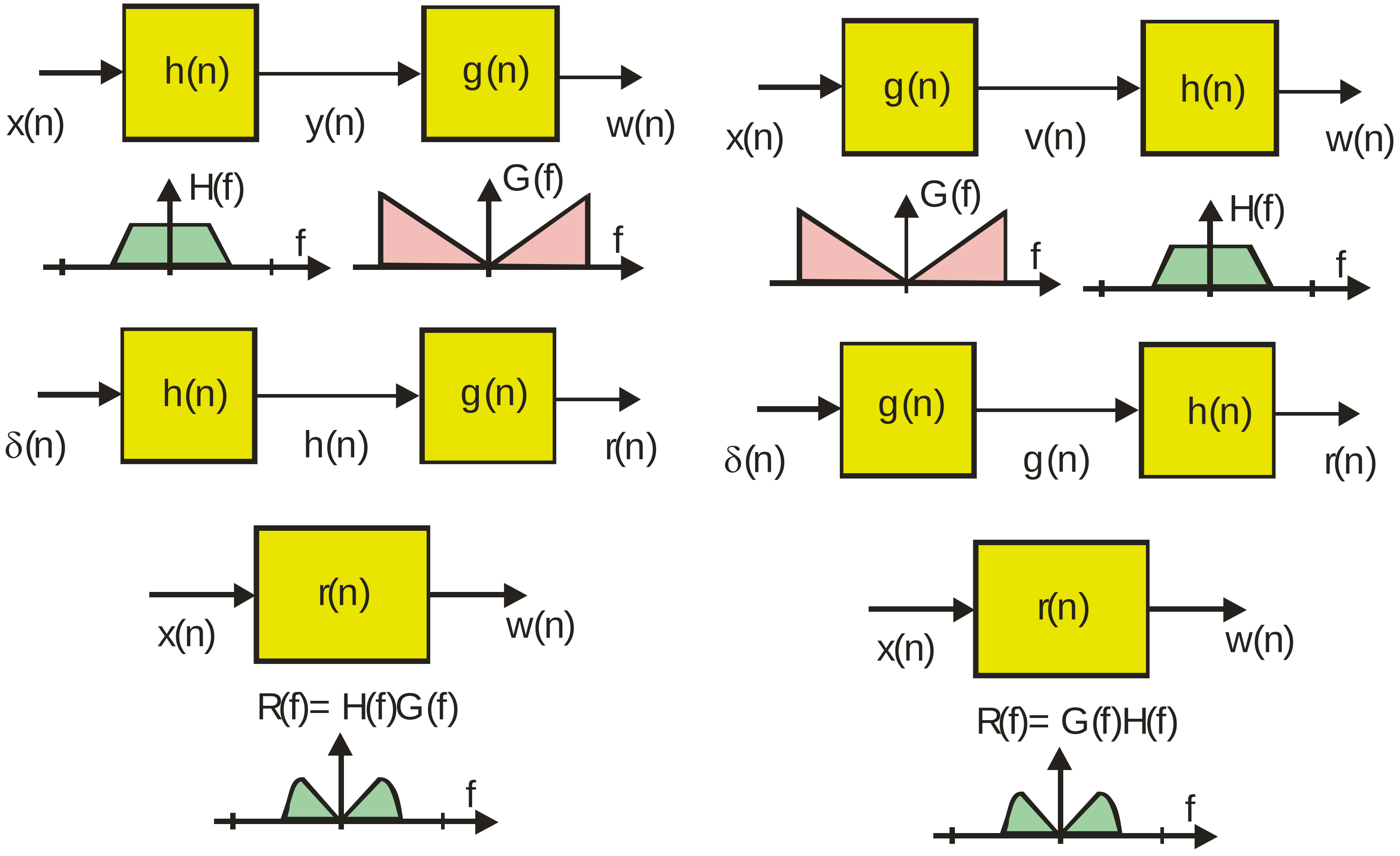
$$\text{or} \quad \theta_k = k \cdot \frac{2\pi}{M}$$

Any Multiple of
Output Sample Rate
Aliases to Baseband

Let's Keep Rearranging the Players!



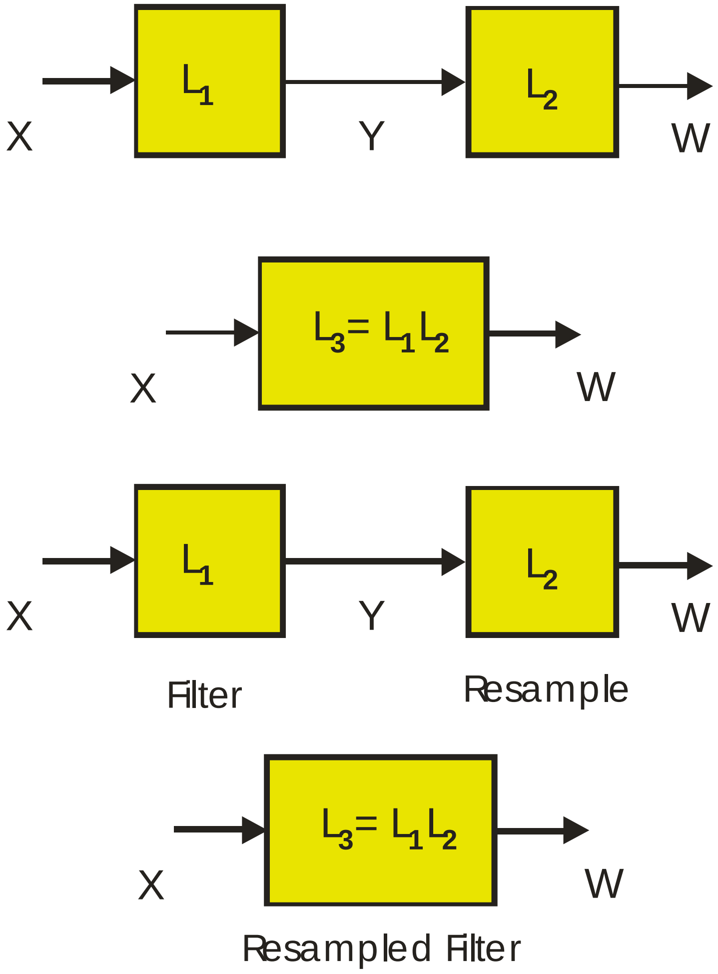
Linear Systems
Commute and are Associative



$R(f) = H(f)G(f)$

$R(f) = G(f)H(f)$

Linear systems Are Associative



L_1 L_2

X Y W

$L_3 = L_1 L_2$

X W

L_1 L_2

X Y W

Filter Resample

$L_3 = L_1 L_2$

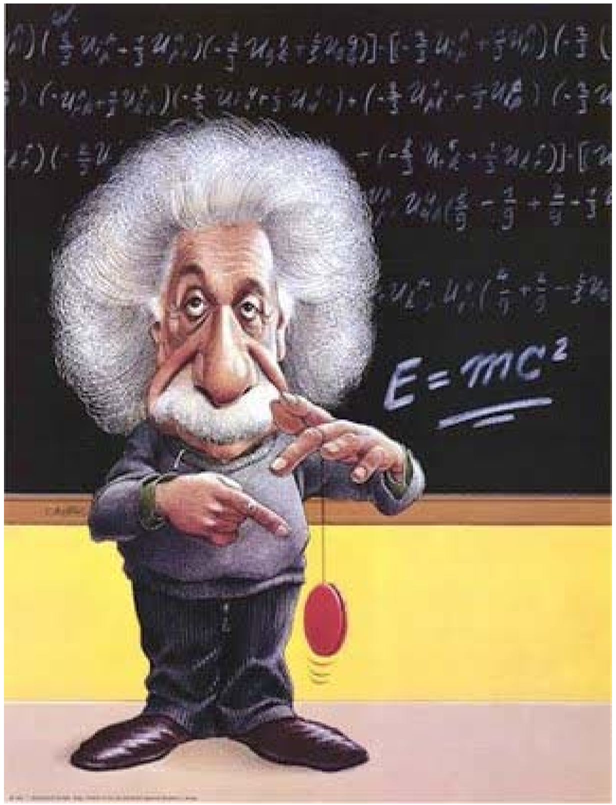
X W

Resampled Filter

In Case you Couldn't Wait to See the Proof

$$y(n) = \sum_{k_1} x(n-k_1)h(k_1)$$

$$dy(n) = \sum_{k_2} y(n-k_2)g(k_2)$$



$$= \sum_{k_2}\sum_{k_1} x(n-k_1-k_2)h(k_1)g(k_2)$$

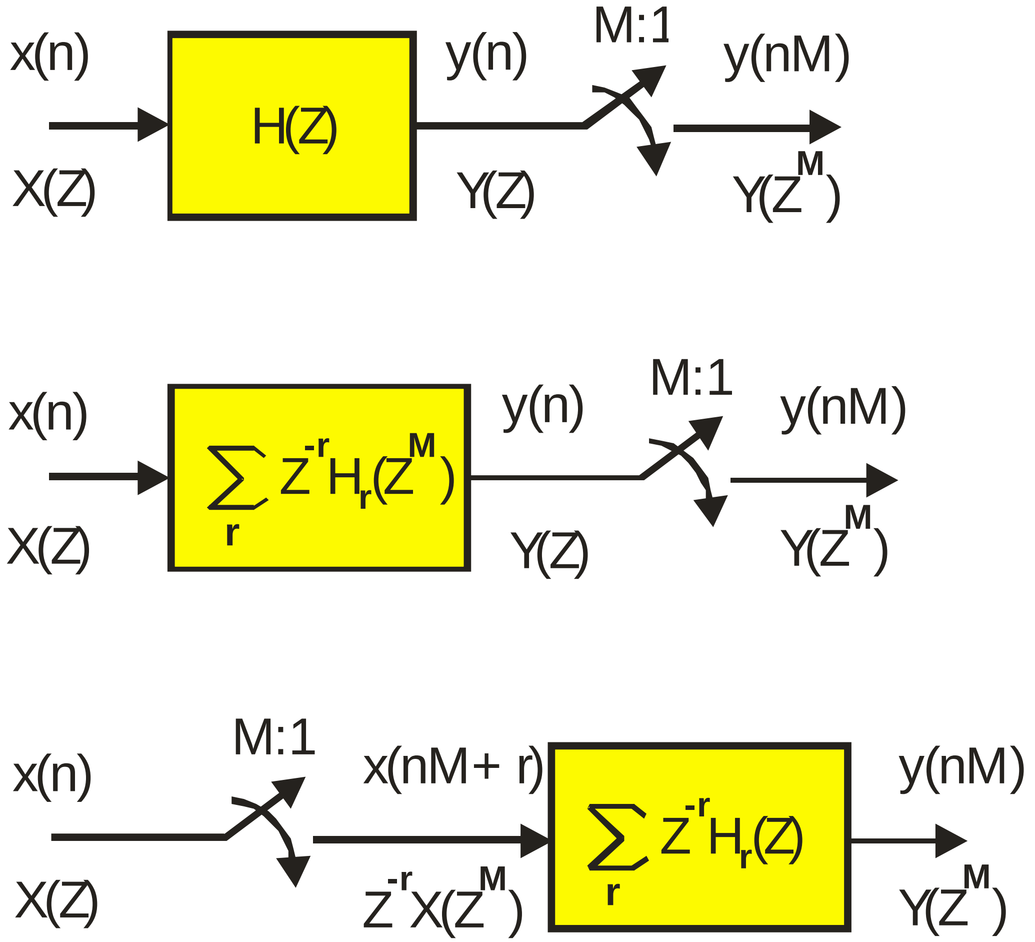
$$= \sum_{k_2}\sum_{k_3} x(n-k_3)h(k_3-k_2)g(k_2)$$

$$= \sum_{k3} x(n-k_3)\sum_{k_2} h(k_3-k_2)g(k_2)$$

$$= \sum_{k_3} x(n-k_3)f(k_3)$$

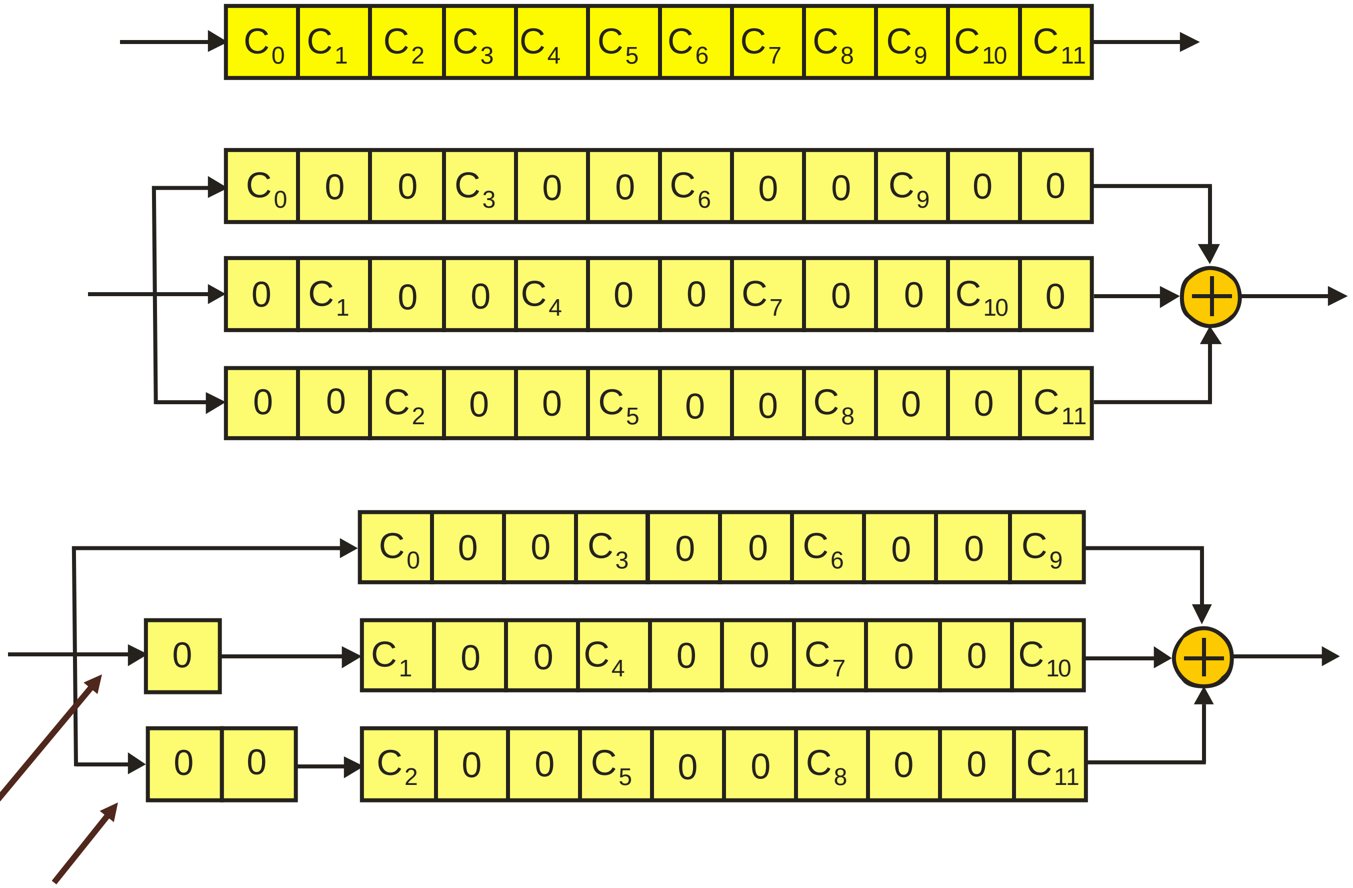
$$\text{where } f(n) = \sum_{k_2} h(n-k_2)g(k_2)$$

Filter and Output Resampler can Commute to Input Resampler and Resampled Filter



Coefficient Assignment of Low-Pass Polyphase Partition

For M-to-1 resample start at Index r and Increment by M
For 3-to-1 resample start at index r and increment by 3



Extract Delays To First
Non-Zero Coefficient

This mapping from 1-D to 2-D is used by Cooley-Tukey FFT.
Polyphase Filters and CT-FFT are kissing cousins!

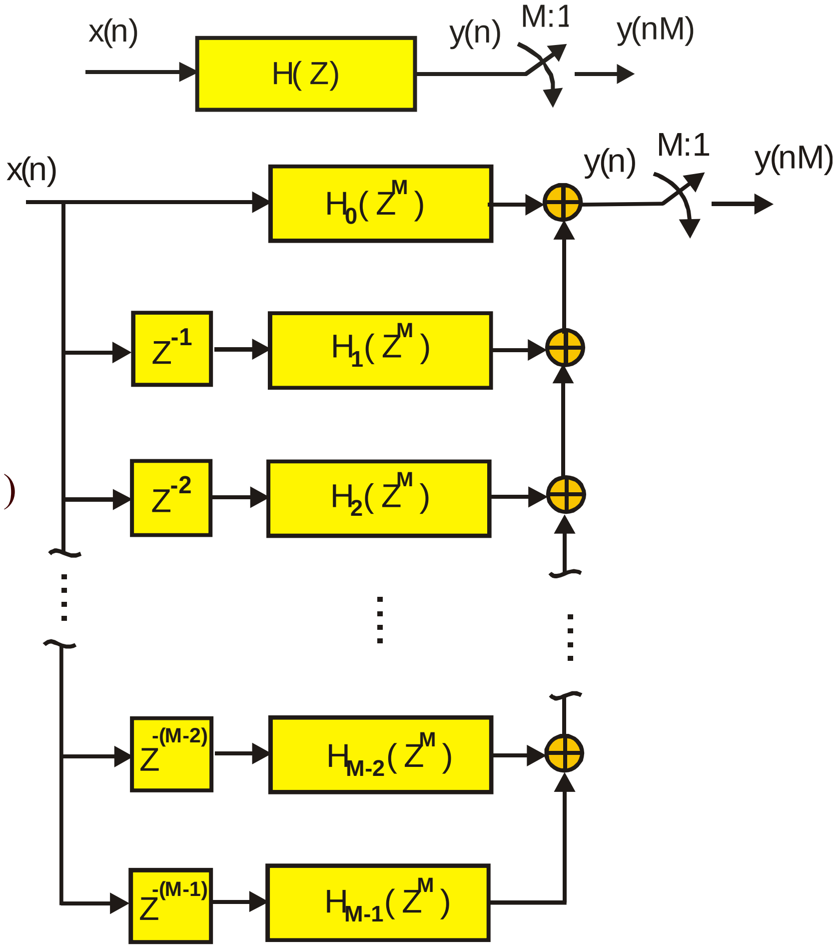
Polyphase Partition of Low Pass Filter

1-Path to M-Path Transformation

$$H(Z)=\sum_{n=0}^{N-1} h(n)Z^{-n}$$

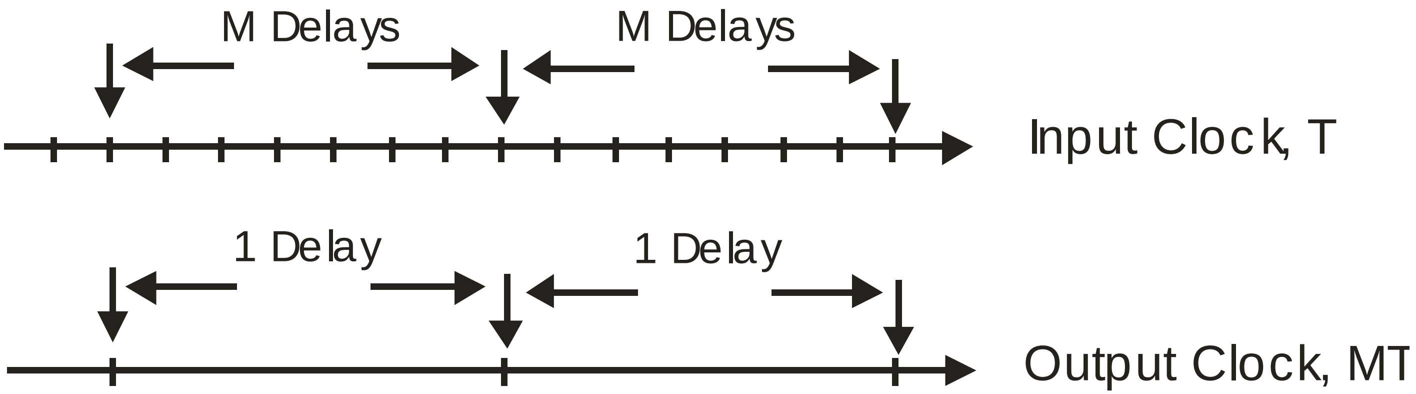
$$H(Z)=\sum_{r=0}^{M-1}\sum_{n=0}^{N-1} h(r+nM)Z^{-(r+nM)}$$

$$H(Z)=\sum_{r=0}^{M-1} Z^{-r}\sum_{n=0}^{N-1} h(r+nM)Z^{-nM}$$

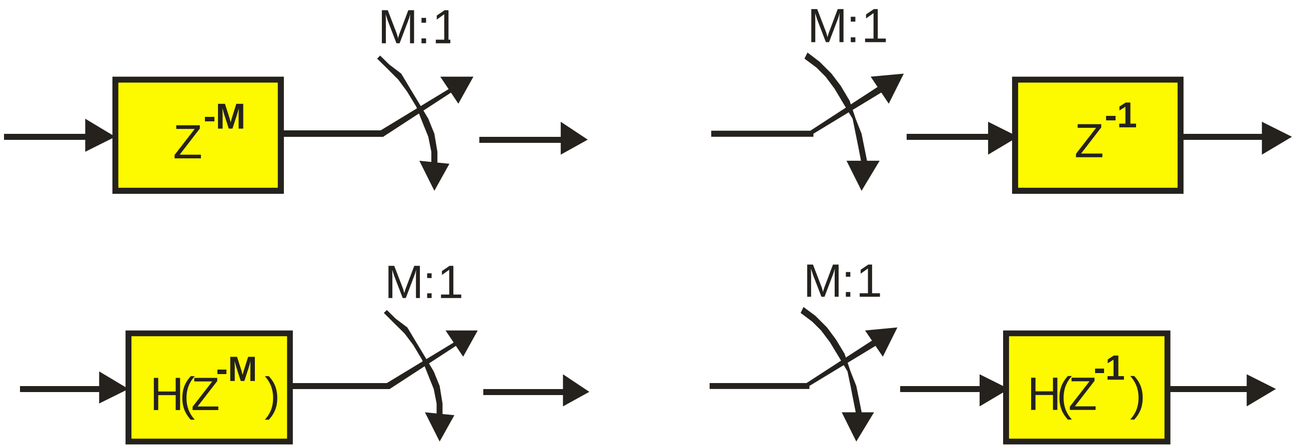


M-Path Partition Supports M-to-1 Down Sample
Also Supports Rational Ratio M-to-Q and M-to-Q/P Down Sample!

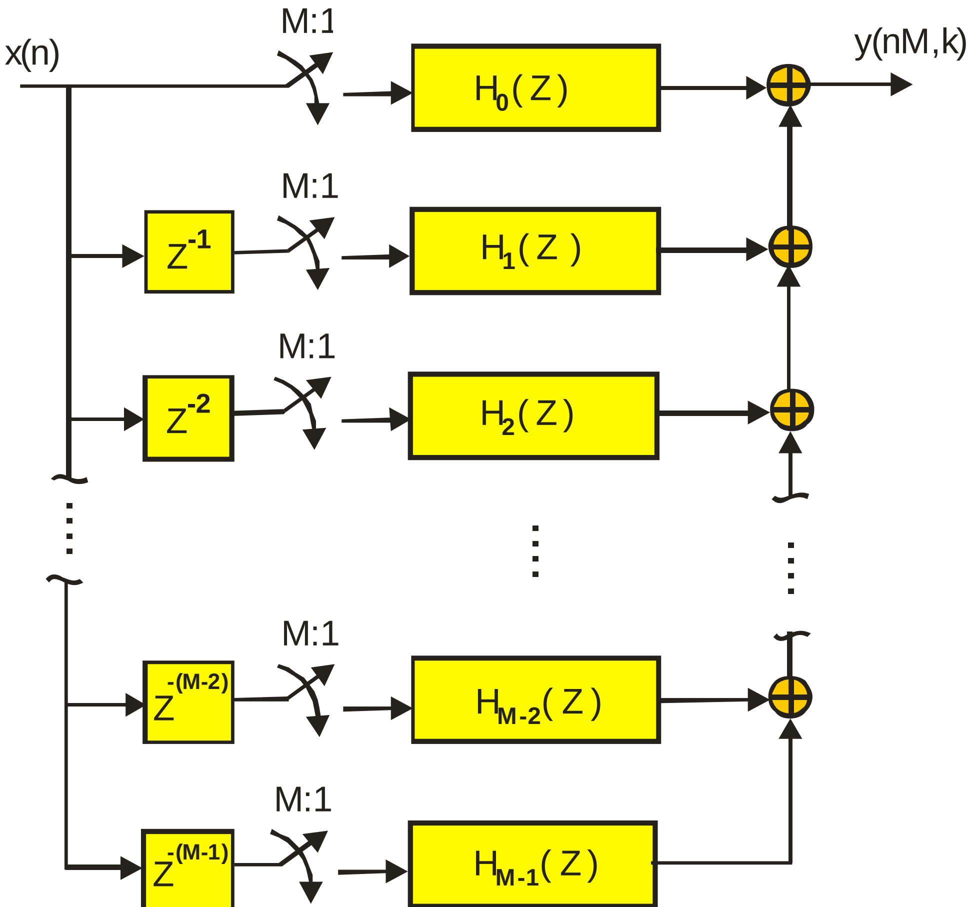
Noble Identity: Commute M-units of Delay followed by M-to-1 Down Sample



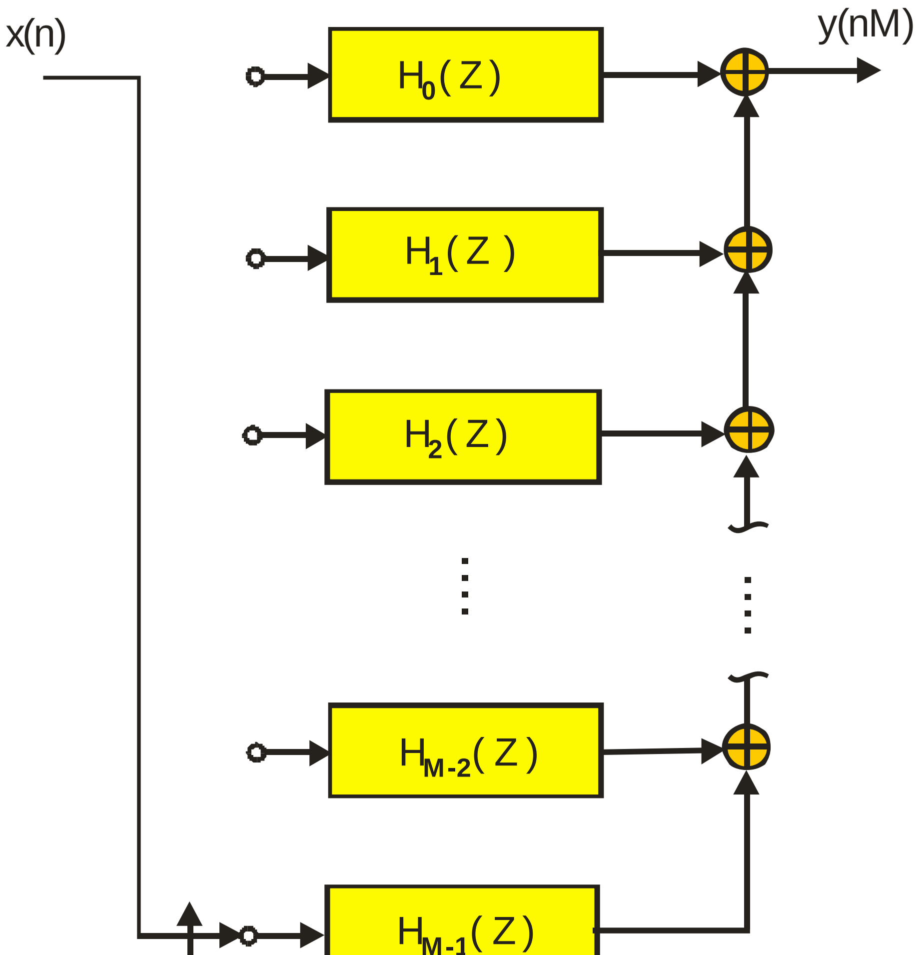
M-Units of Delay at Input Rate Same as 1-Unit of Delay at Output Rate

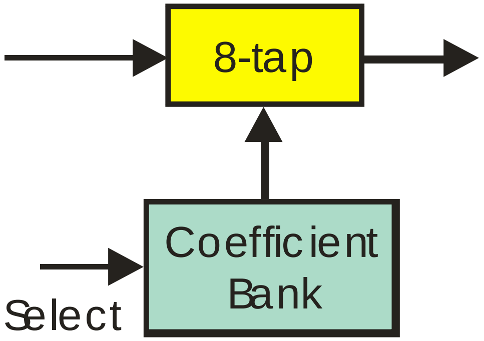


Interchange Filters and Resampler: Place Resampler at Input Rather Than at Output of Filter



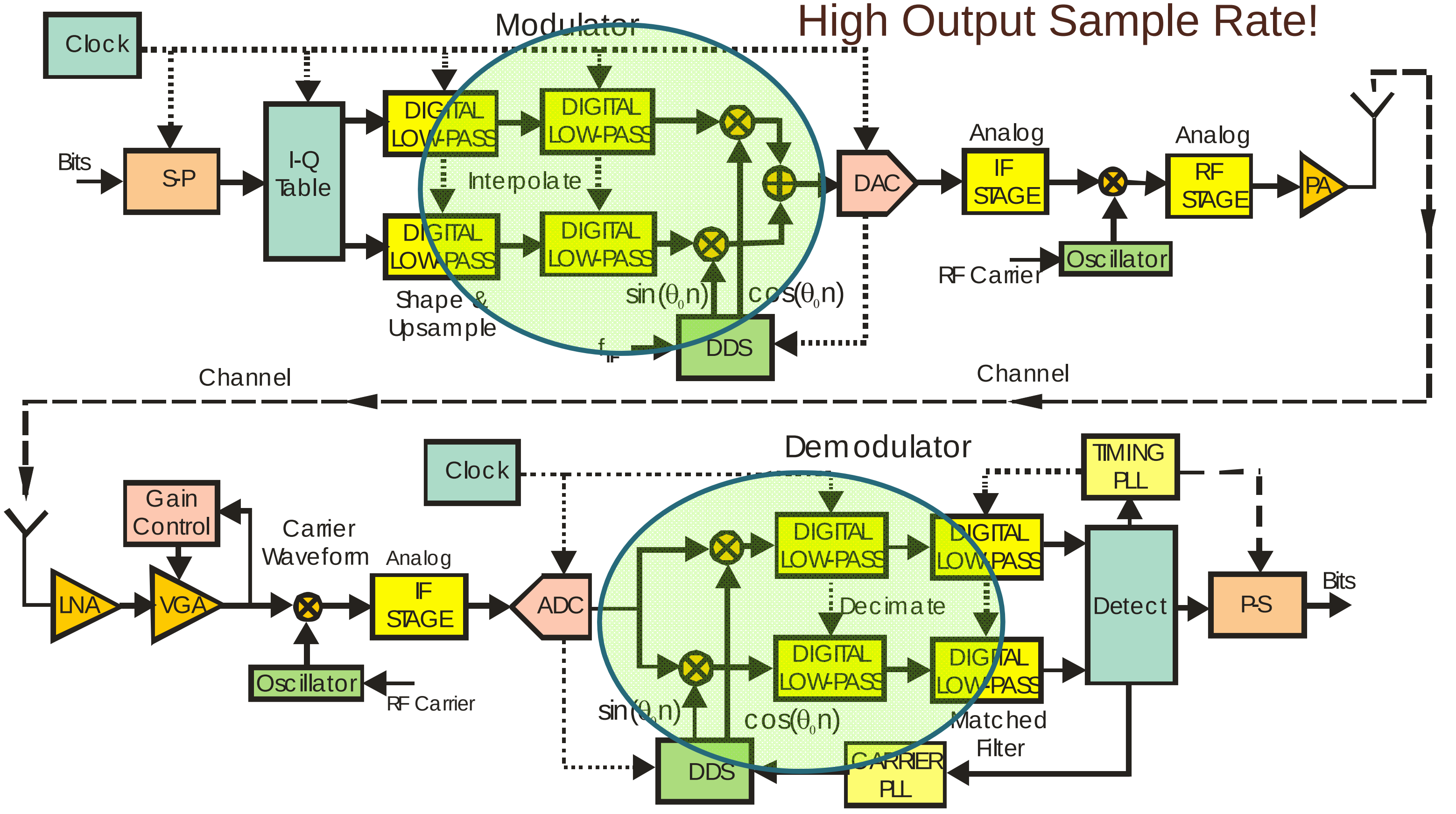
Replace Delays with Commutator
Perform Path Operations Sequentially





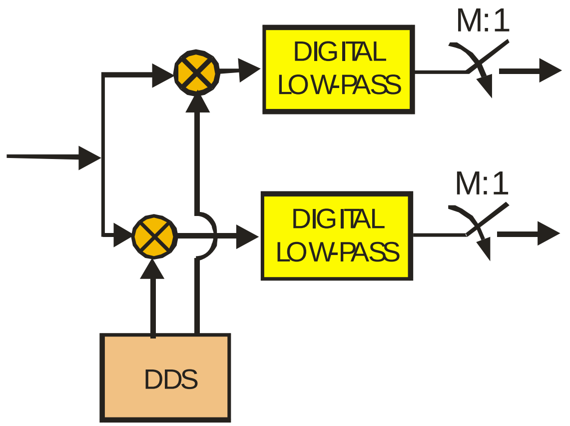
Transmitter Process, Up-Sample and Up-Convert
Receiver Process, Down Convert and Down-Sample

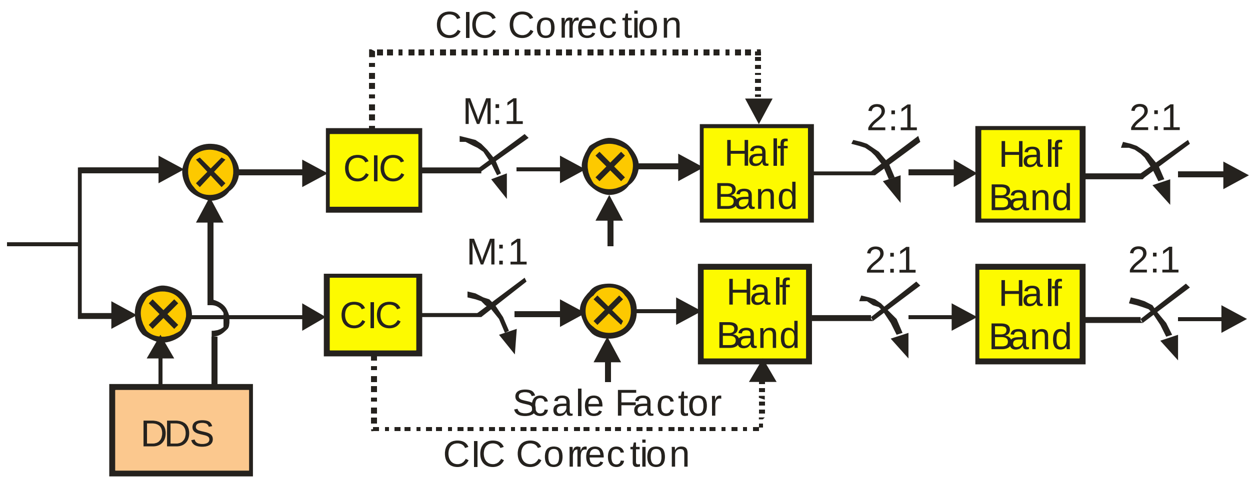
Modulator Raises Sample Rate &
Applies Heterodyne at
High Output Sample Rate!

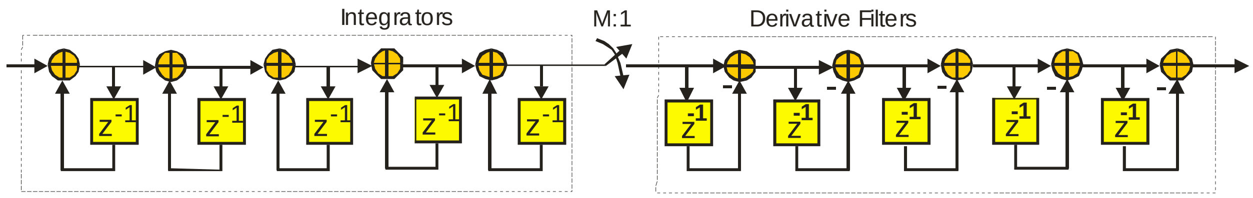


De-Modulator Applies Heterodynes
at High Input Rate & then Reduces

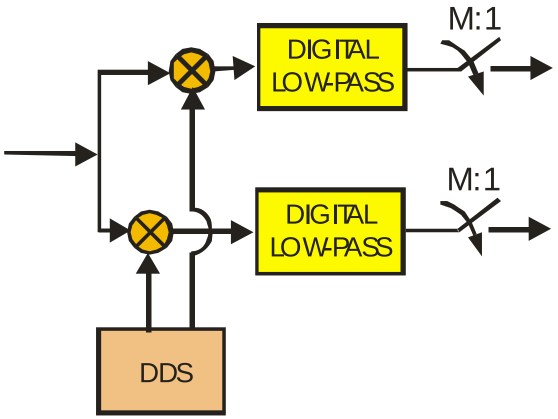
Conventional and Ubiquitous DDC

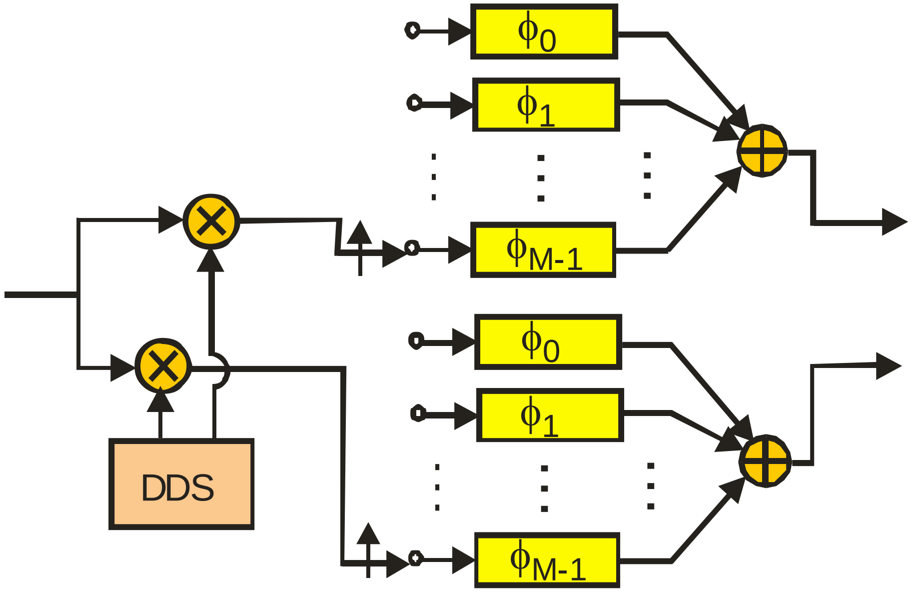


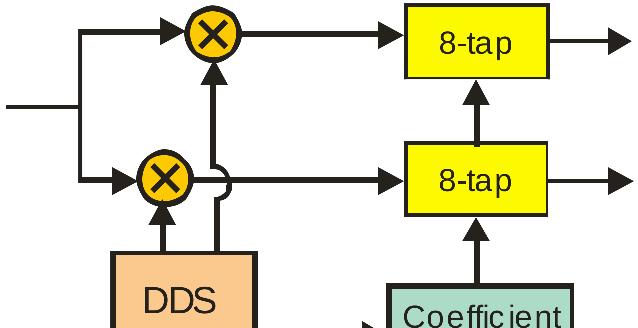




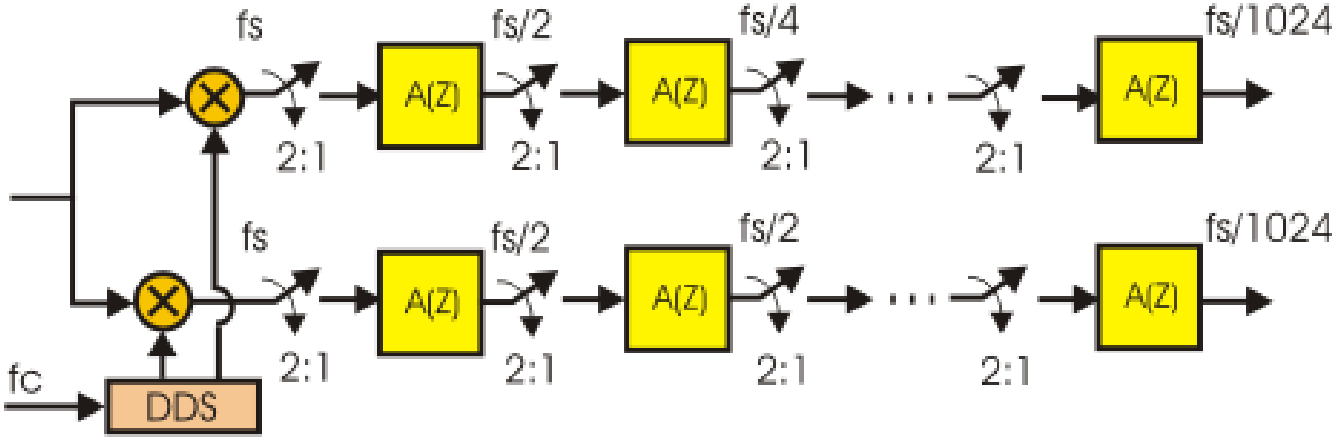
Convert Two Parallel Paths into M Sequential Paths for each Path





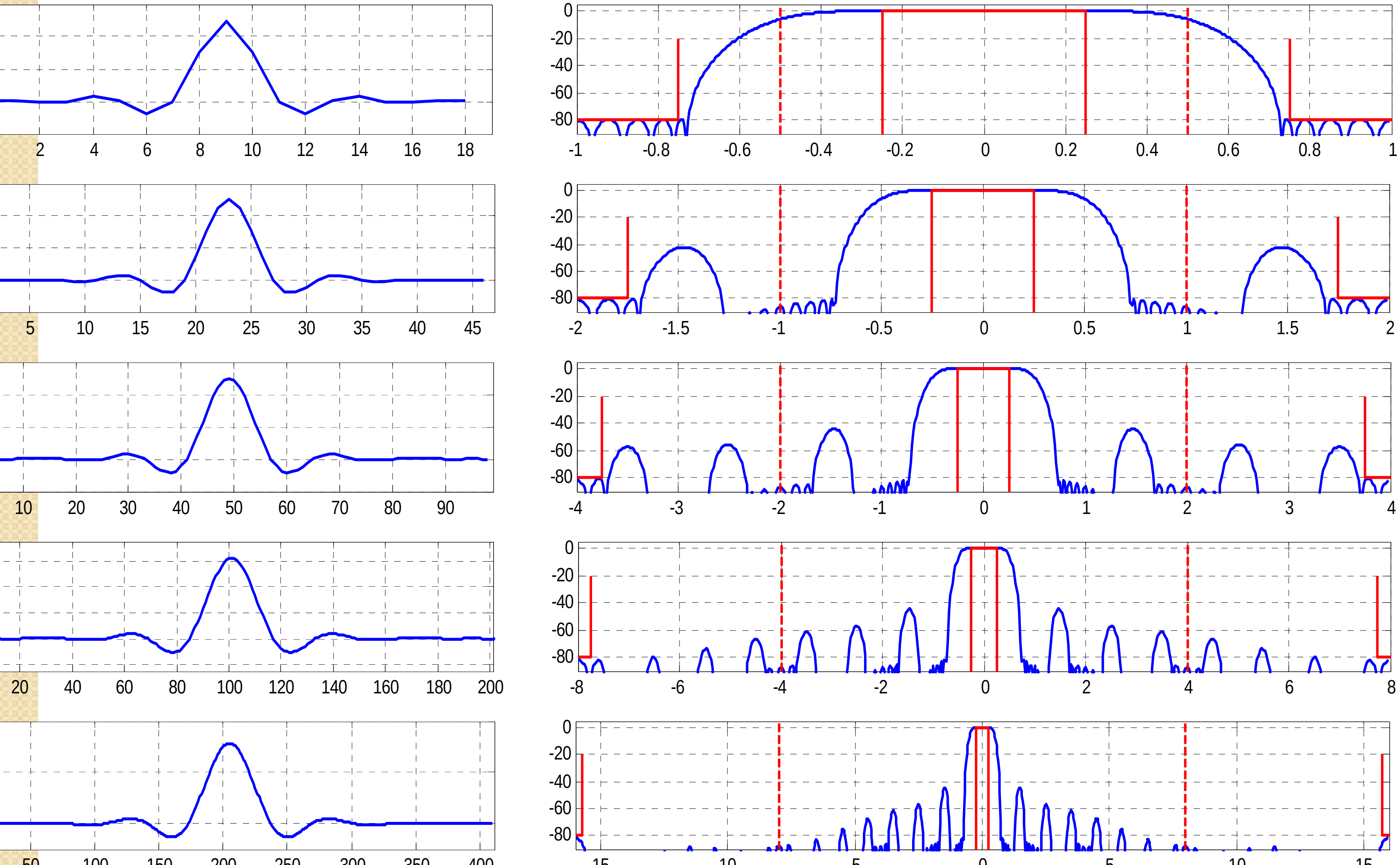


Replace CIC with
Cascade 2-to-1 Half Band FIR Filters

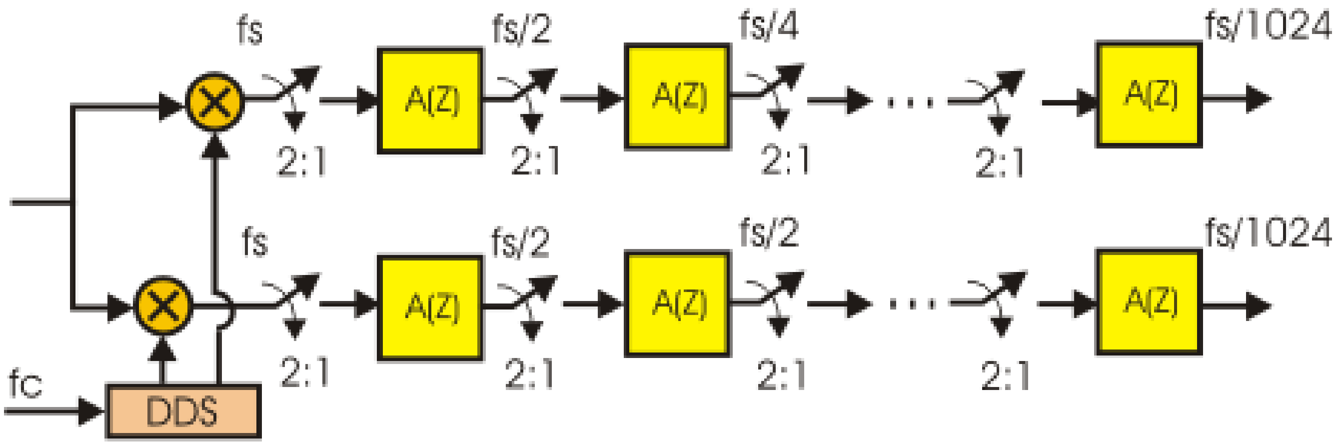


Filter Number	1	2	3	4	5	6	7	8	9	10	Total
Number Taps	3	3	3	3	7	7	7	7	11	19	70
Operations Per Filter	2-A 2-Shifts	2-A 2-Shifts	2-A 2-Shifts	2-A 2-Shifts	4-A 2-Mult	4-A 2-Mult	4-A 2-Mult	4-A 2-Mult	6-A 3-Mult	10-A 5-Mult	___
Adds Ref to Input	2	2/2	2/4	2/8	4/16	4/32	4/64	4/128	6/256	10/512	4.26
Mult Ref to Input	0	0	0	0	2/16	2/32	2/64	2/128	3/256	5/512	0.27

Impulse and Frequency Response of Last Stage Referred to Earlier Stages

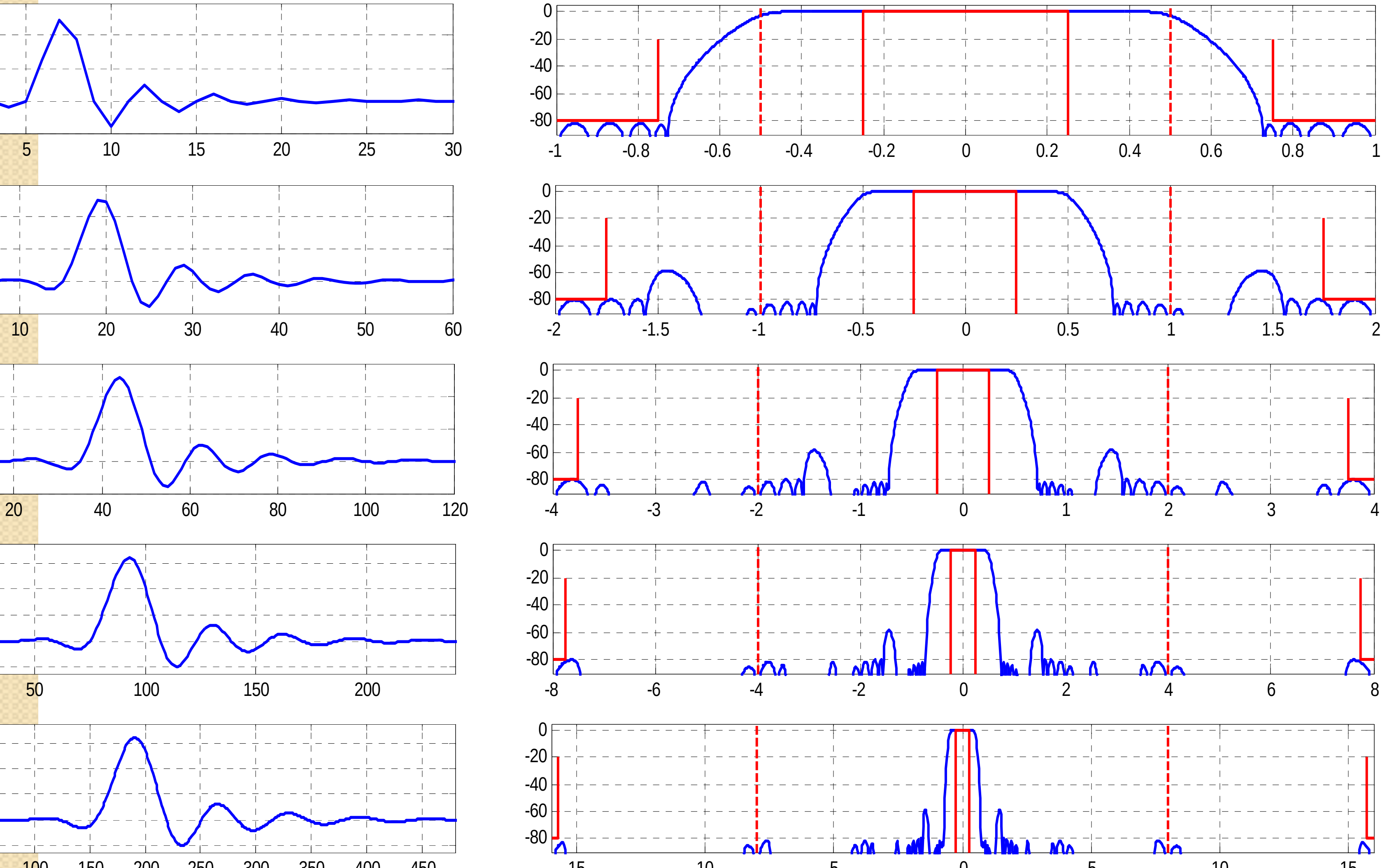


Replace CIC with Cascade 2-to-1 Half Band Linear Phase IIR Filters



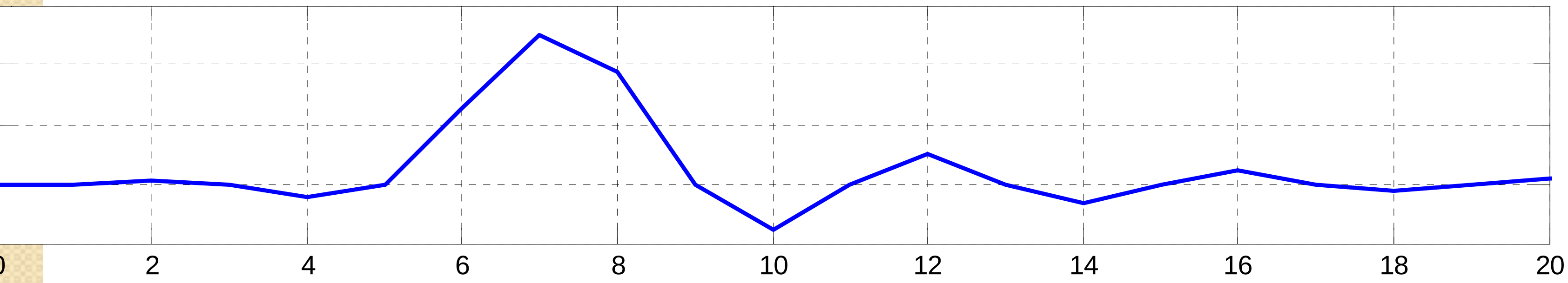
Filter Number	1	2	3	4	5	6	7	8	9	10	Total
Number Taps	1	1	1	1	3	3	3	3	3	4	23
Operations Per Filter	3-A 1-Mult	3-A 1-Mult	3-A 1-Mult	3-A 1-Mult	7-A 3-Mult	7-A 3-Mult	7-A 3-Mult	7-A 3-Mult	7-A 3-Mult	9-A 4-Mult	___
Adds Ref to Input	3/2	3/4	3/8	3/16	7/32	7/64	7/128	7/256	7/512	9/1024	3.25
Mult Ref to Input	1/2	1/4	1/8	1/16	3/32	3/64	3/128	3/256	3/512	4/1024	1.12

Impulse and Frequency Response of Last Stage Referred to Earlier Stages

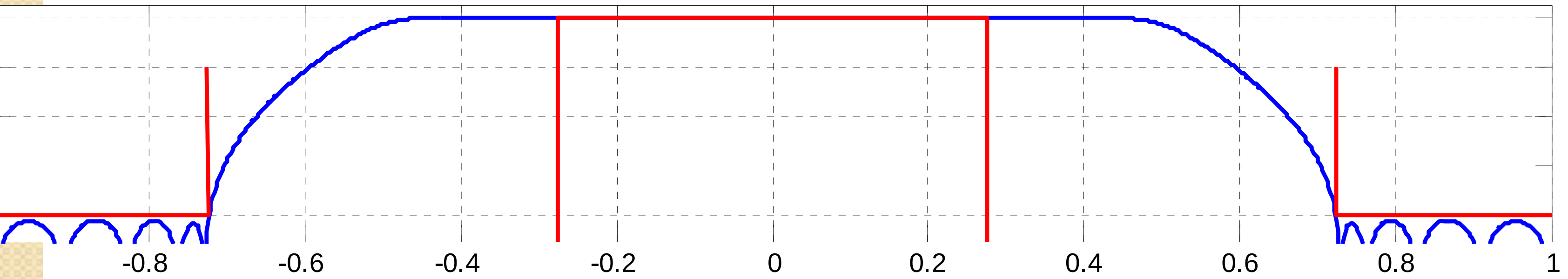


Impulse, Frequency, & Group Delay Response of 2-Path Linear Phase, Recursive Half-Band Filter

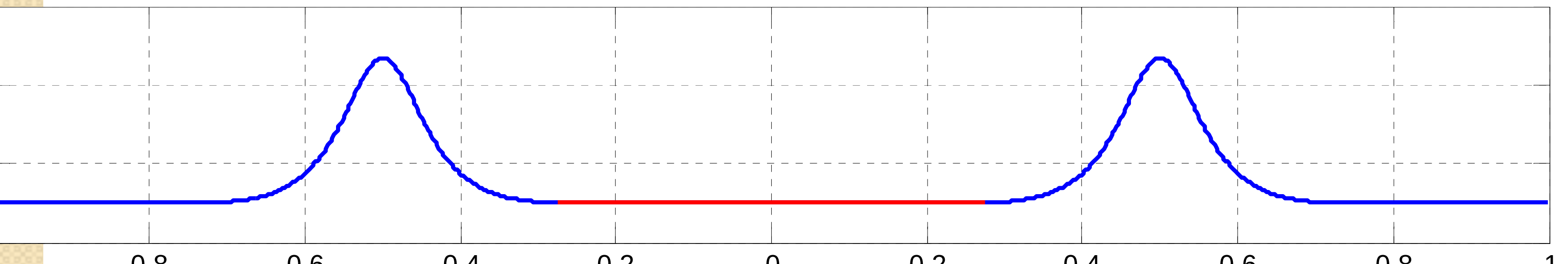
Impulse Response



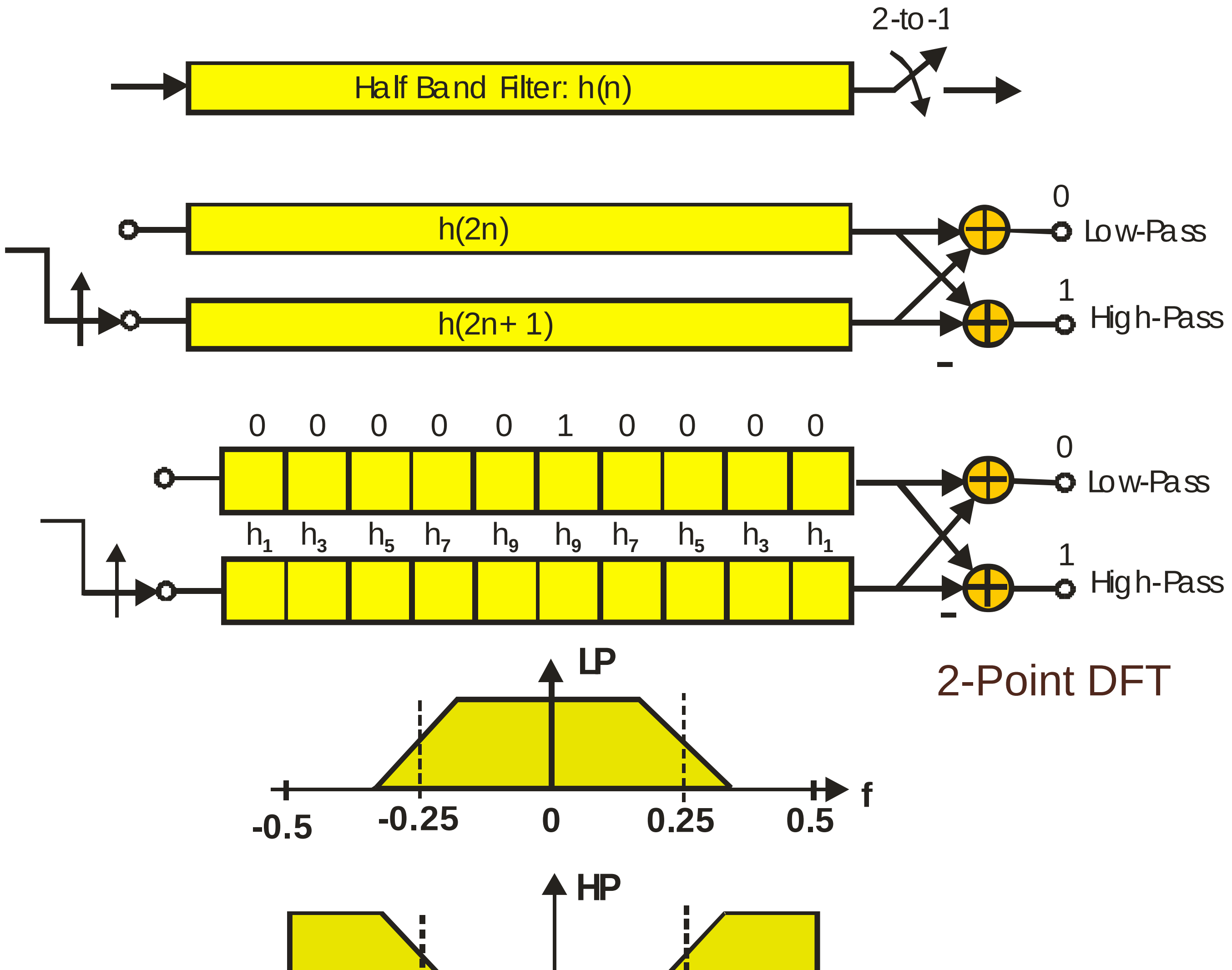
Frequency Response



Group Delay

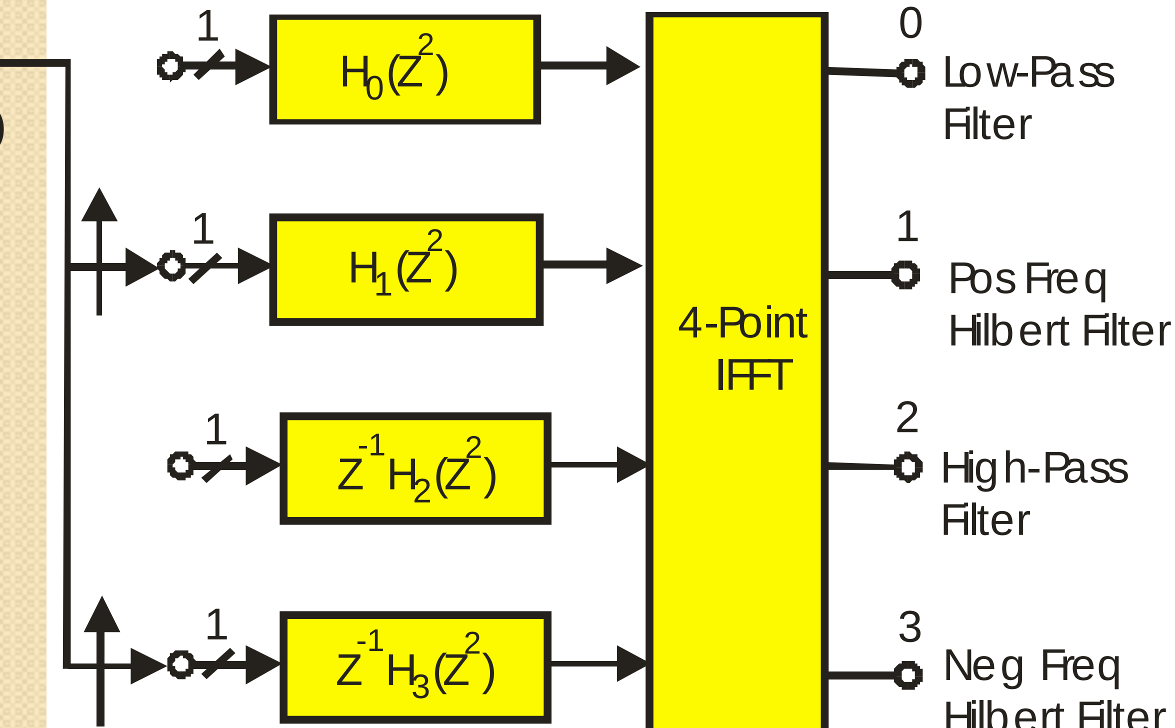


2-to-1 Resampling 2-Path Polyphase Filter and Digital Down-Converter



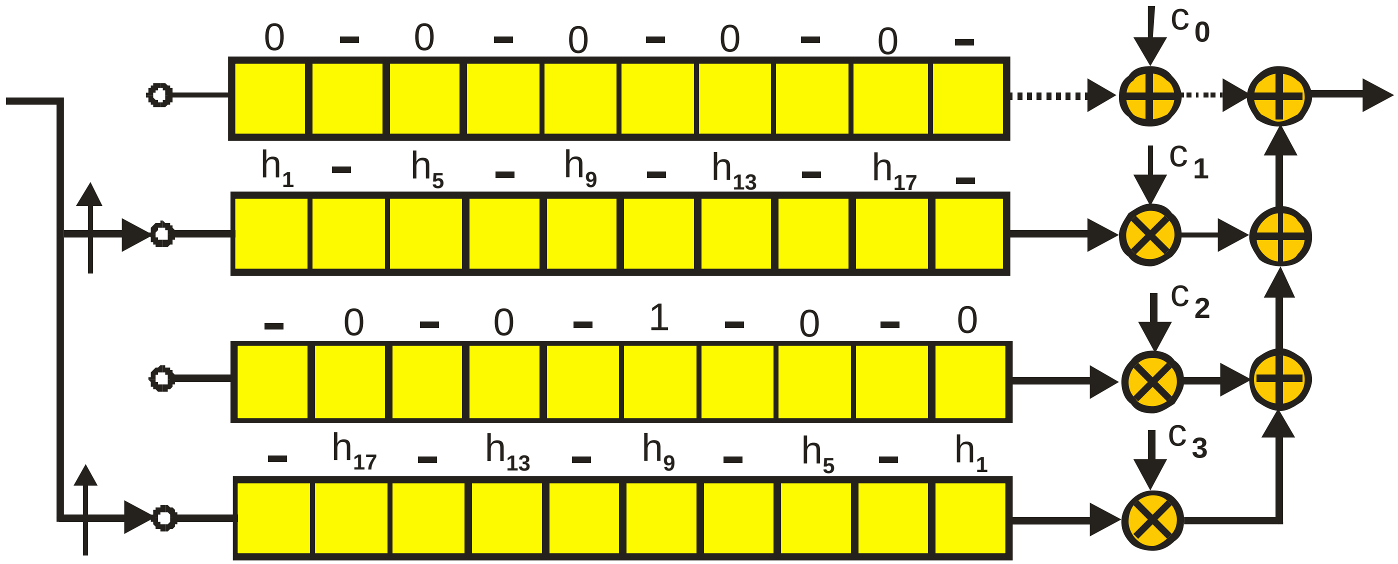
2-Point DFT

o-1Resampling 4-Path Down-Sample Polyphase Filter and 4-Point IFFT Extracts Signal Component From One-of-Four Selected Nyquist Zones



Half Band Filters
Centered on
Cardinal Directions
Each Reduces
BW 2-to-1
and Reduces
Sample Rate 2-to-1

4-Path, 2-to-1 Down-Sample with 4 Possible Trivial Phase Shifters

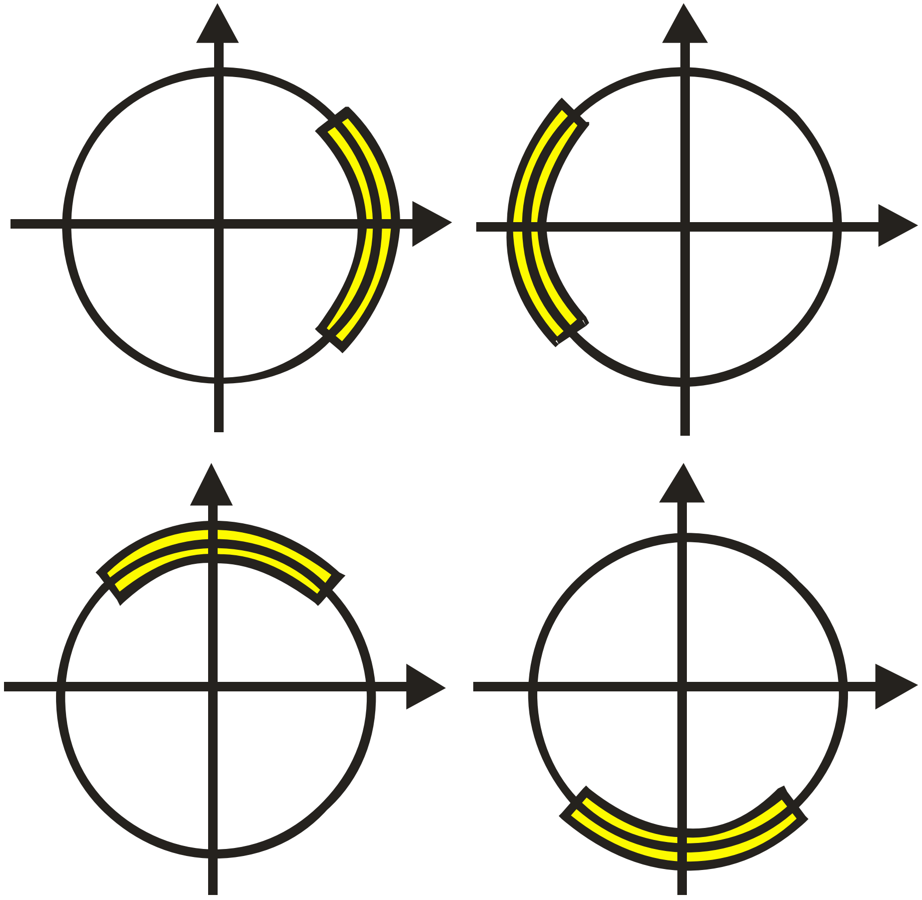


2-to-1
own-Sample

4-Path Polyphase Filter
Path-0 Not Used
2.5-Multiplies per Input

4-Phase Rotators
fs·k/4: {c_0 c_1 c_2 c_3}
fs·0/4: {1 1 1 1}
fs·1/4: {1 j -1 -j}
fs·2/4: {1 -1 1 -1}
fs·3/4: {1 -j -1 j}

Four Bands Centered on the Cardinal Directions

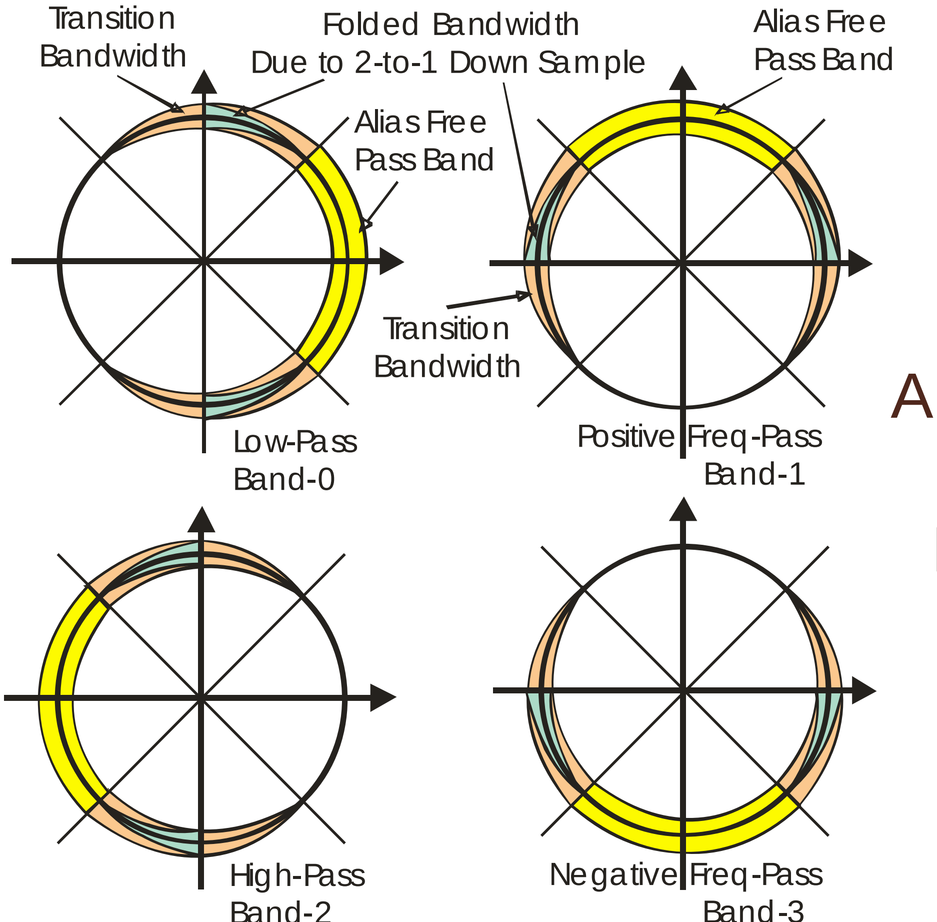


Bands Centered
on 0° and 180 °
(DC and $f_S/2$)
Alias To DC When
Down-Sampled
2-to-1

Bands Centered
on +90° and -90 °
(+$f_S/4$ and –$f_S/4$)
Alias To fs/2 When
Down-Sampled
2-to-1

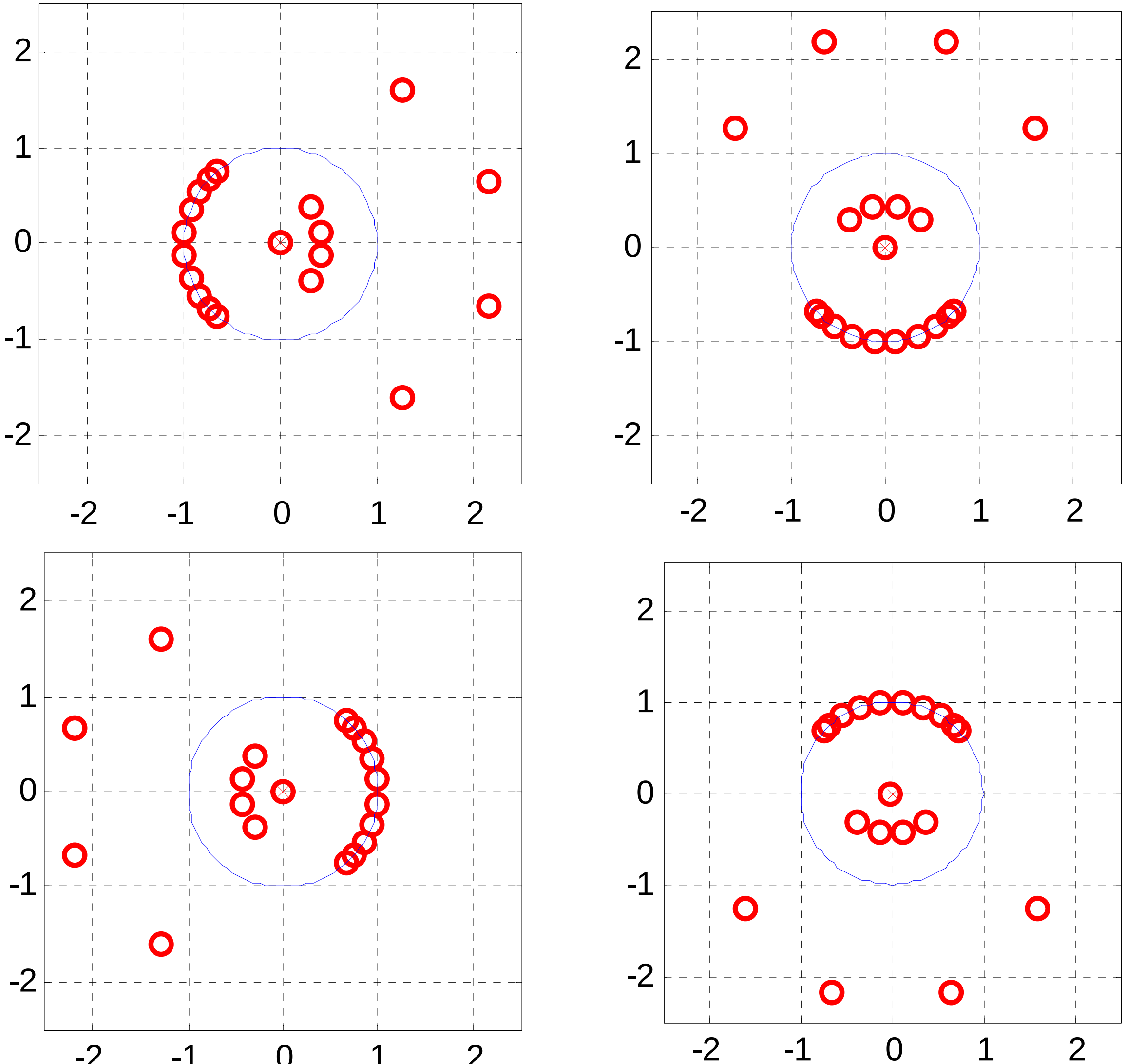
Spectra: Four Half Band Filters on Unit Circle

Showing Alias Free Pass, Transition, and Aliased Bands

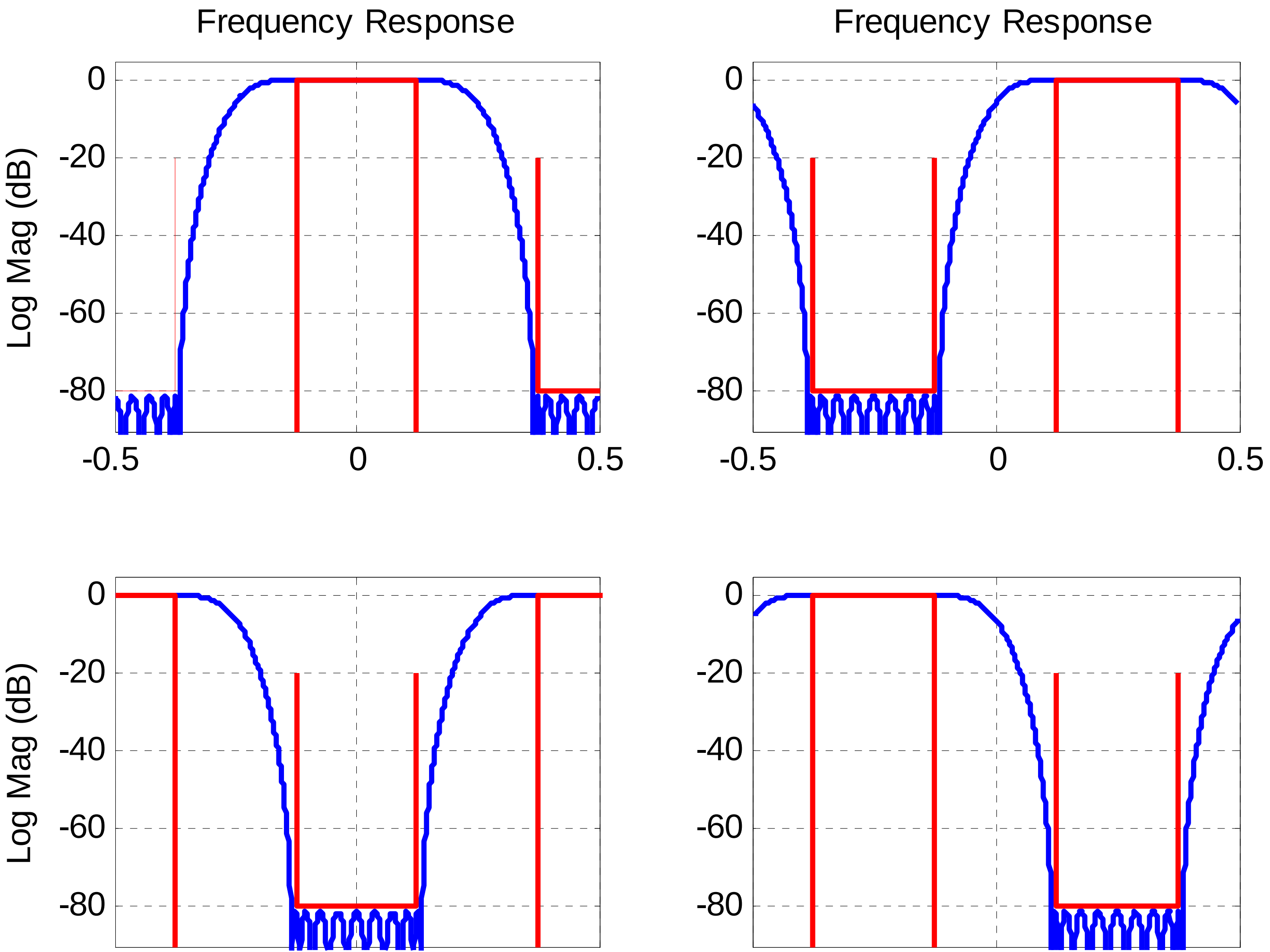


Any Narrowband
Signal Must Reside
in One of the 4
Alias Free Band Intervals.
The Alias Free
Band Intervals Overlap!

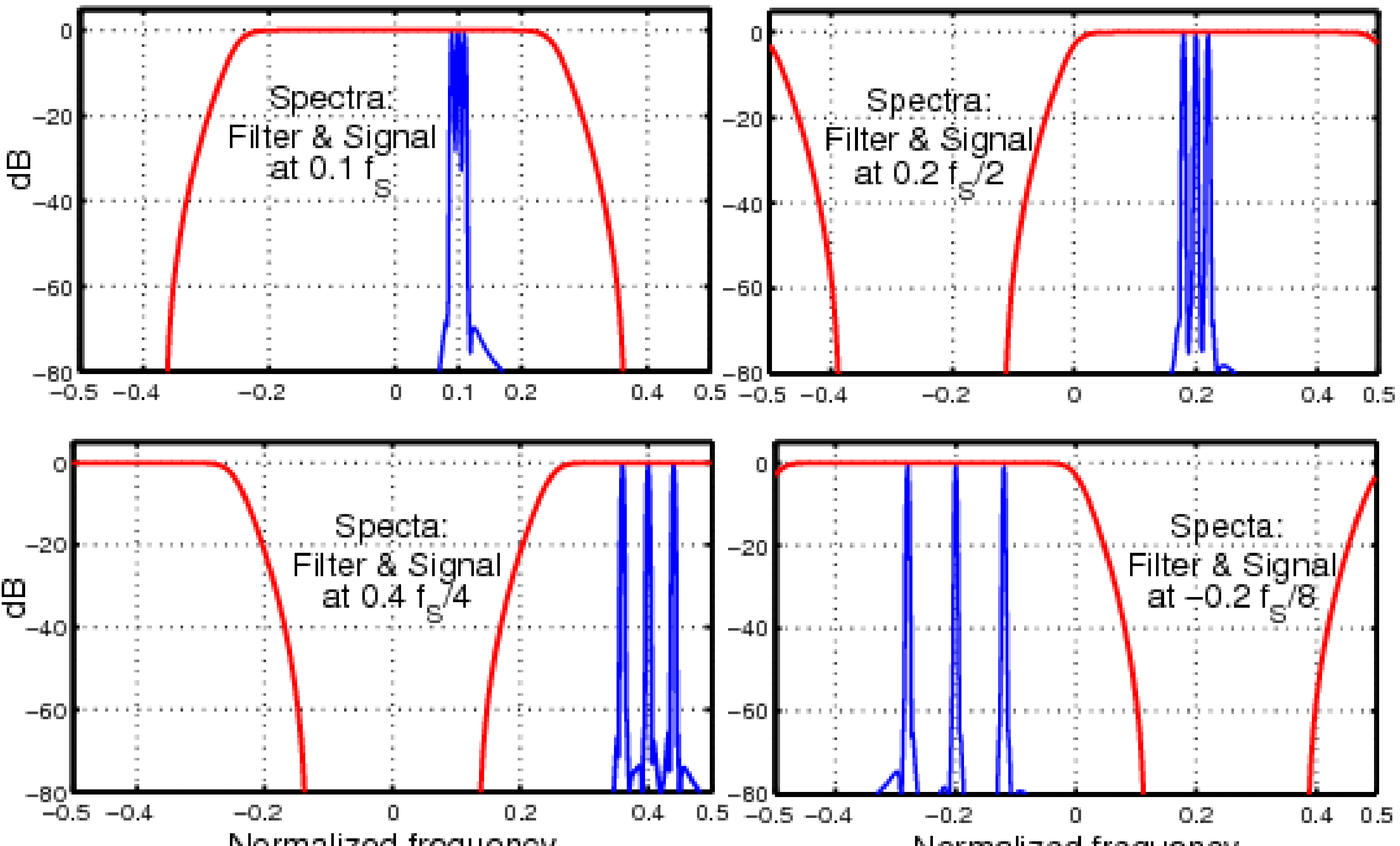
Pole-Zero Diagrams of Four Nyquist Zone Filters



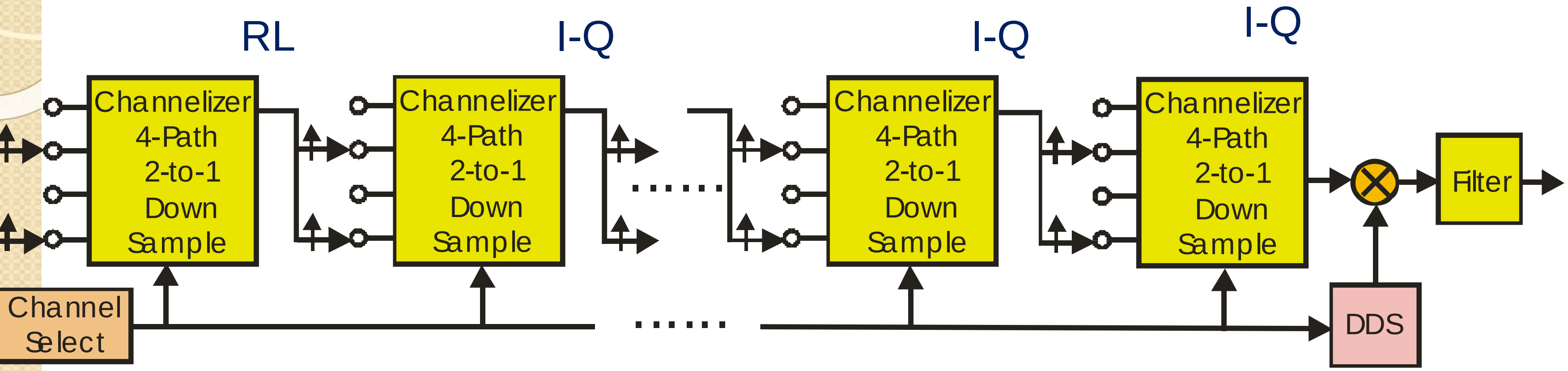
Frequency Responses of Four Nyquist Zone Filters



ctra of Signal Aliased to Different Sampled Data Frequencies in Successive 2-to-1 Sample Rate Reductions.



Most Efficient Multistage Half-Band Digital Down-Converter



RL

I-Q

I-Q

I-Q

Channelizer 4-Path 2-to-1 Down Sample

Channelizer 4-Path 2-to-1 Down Sample

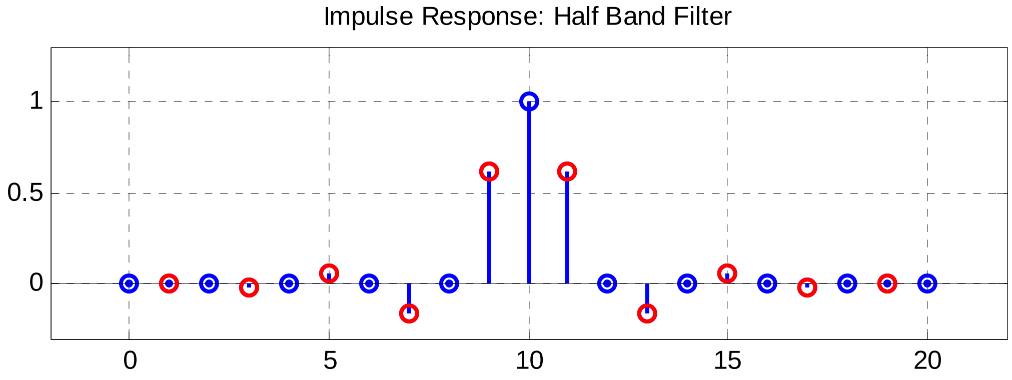
Channelizer 4-Path 2-to-1 Down Sample

Channelizer 4-Path 2-to-1 Down Sample

Filter

DDS

Channel Select



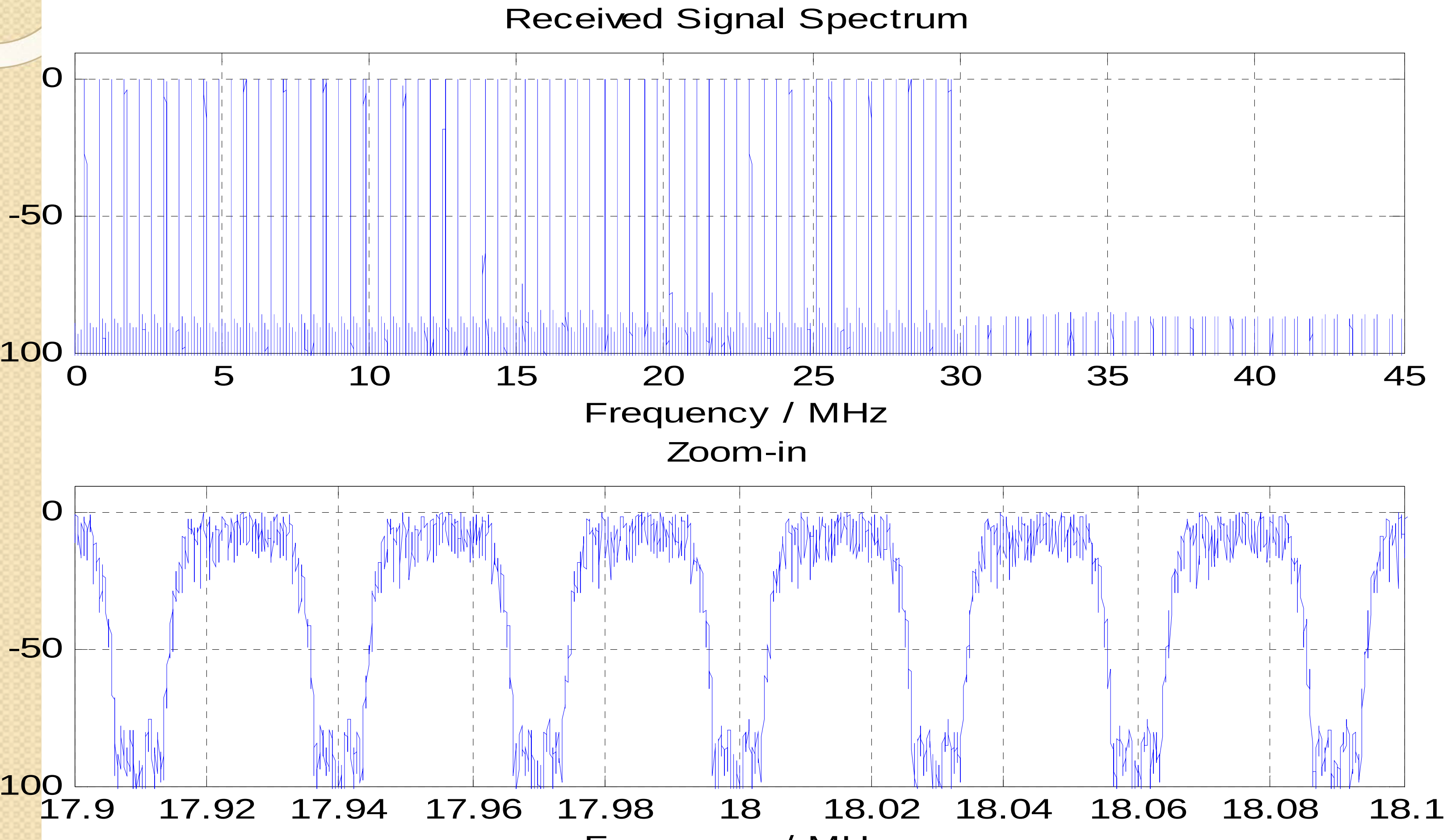
Impulse Response: Half Band Filter

$$N = 2.5 \cdot [1 + \frac{2}{2} + \frac{2}{4} + \frac{2}{8} + \cdots]$$

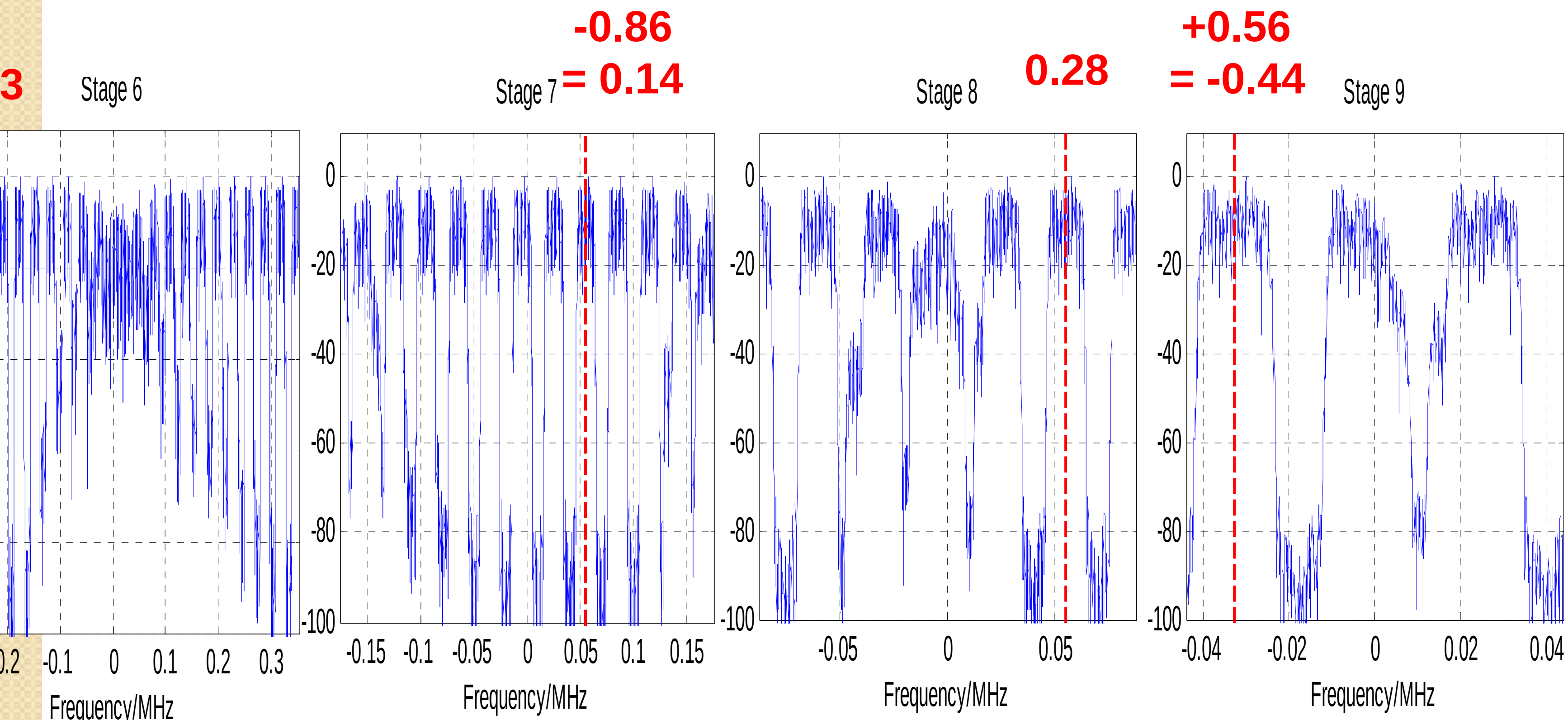
$$= 2.5 + 2.5 \cdot [1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots]$$

$$< 2.5 \cdot 3 = 7.5$$

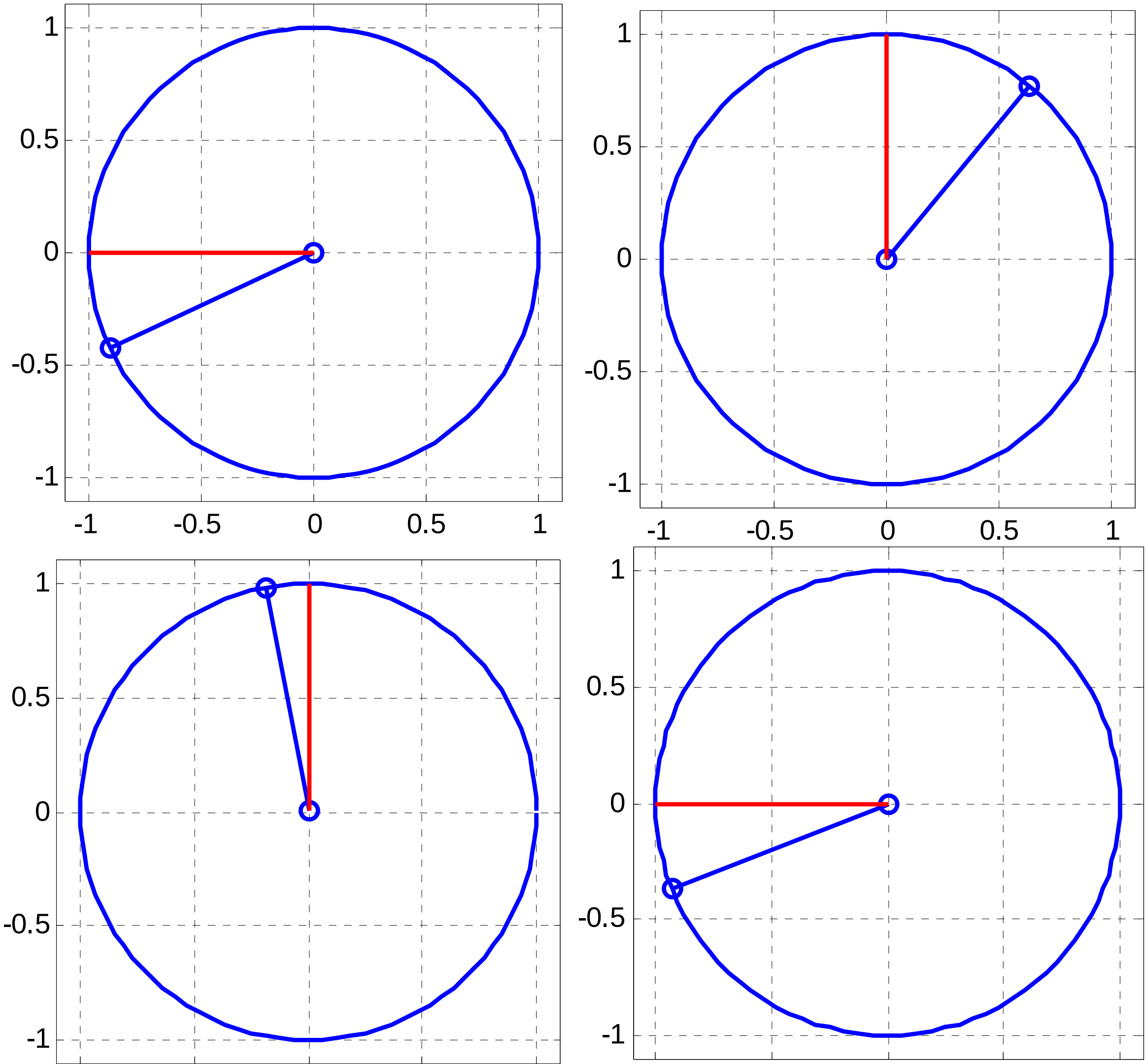
Spectrum of Input Signal and Zoom to Spectral Segment



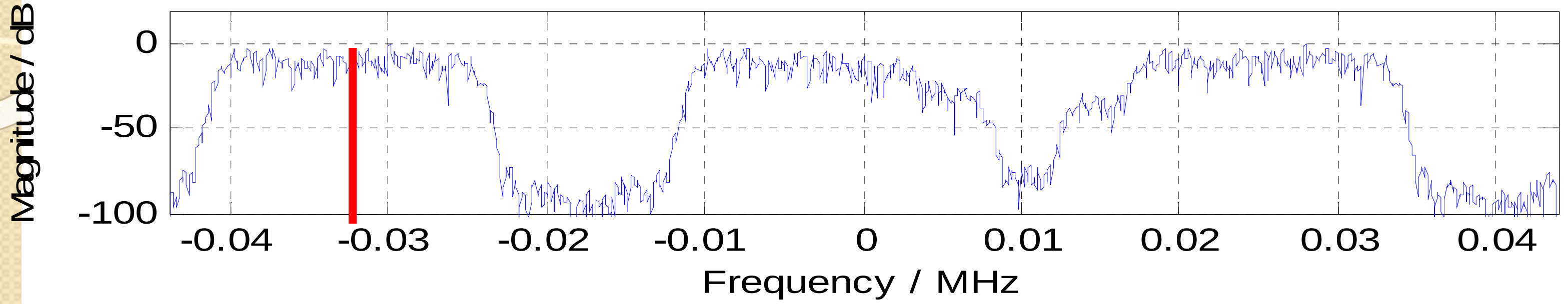
Spectra: Last Four Stages Processing Chain. Dotted Line Indicates Center Frequency of Desired Spectral Component

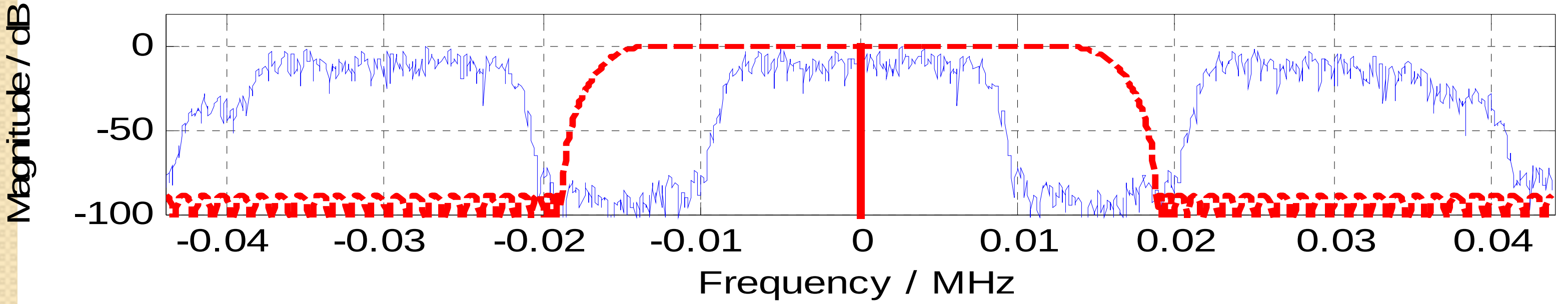


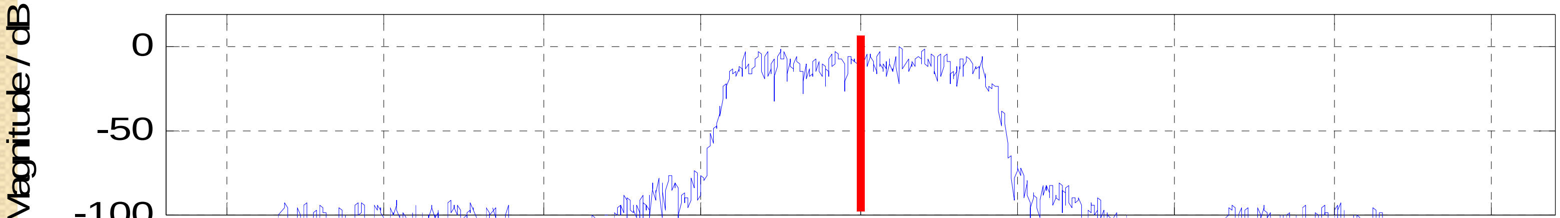
ampled Data Frequency Locations on Successive Aliases



pectrum at Input and Output of Final Heterodyne and Filter Stage


Post Processing--Input Signal Spectrum (ch # 600)

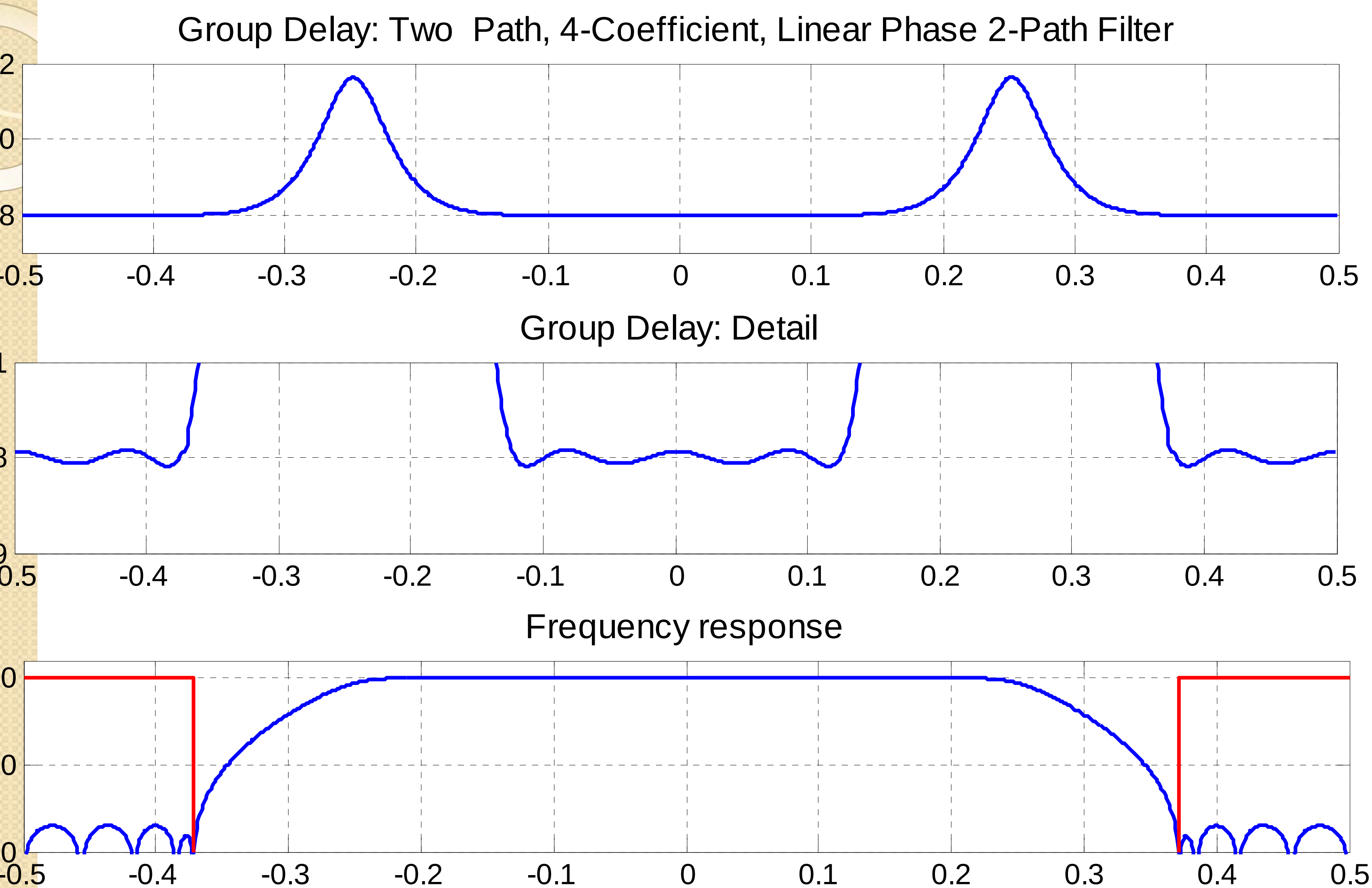

Post Processing--Down Convert (ch # 600)


Post Processing--Filtering (ch # 600)

A 375-to-1 down-sample:
90 MHz to 240 kHz with a 30 kHz output BW 80 dB dynamic range.
Require 6 CIC stages. The gain of each stage is 375:
Gain of 6 stages becomes $(375)^6$ or $2.8 \cdot 10^{15}$ or
52 bits growth in the CIC integrators.
With 16-bit input data integrator bit width is 16+52 or 68.
Six integrators in both I & Q paths would be circulating 816 bits per input sample which if converted to the 16-bit width required of the arithmetic in the half-band filters proves to be same number of bits to manipulate 48 arithmetic operations per input sample.

Number of operations for the I-Q half band filter chain is on the order of 8-multiply and 16 adds per input sample which represents a workload 1/6 of the CIC chain. The efficient cascade CIC filter chain can be replaced with an even more efficient cascade four-path half band filter chain.

Linear Phase IIR Filter

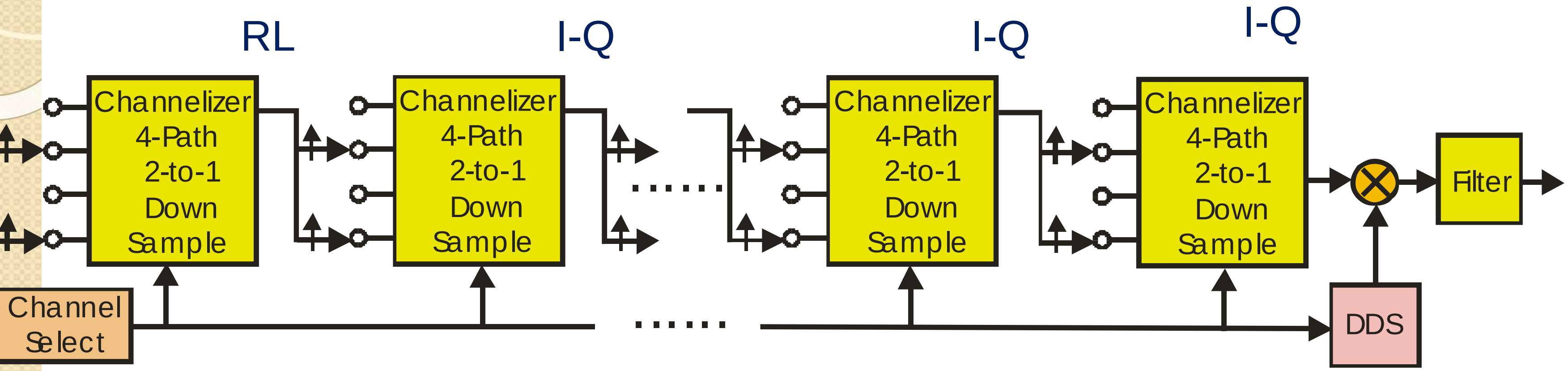


Group Delay: Two Path, 4-Coefficient, Linear Phase 2-Path Filter

Group Delay: Detail

Frequency response

Most Efficient Multistage
Half-Band Digital Down-Converter


RL
I-Q
I-Q
I-Q

Channelizer
4-Path
2-to-1
Down
Sample

Channelizer
4-Path
2-to-1
Down
Sample

……

Channelizer
4-Path
2-to-1
Down
Sample

Channelizer
4-Path
2-to-1
Down
Sample

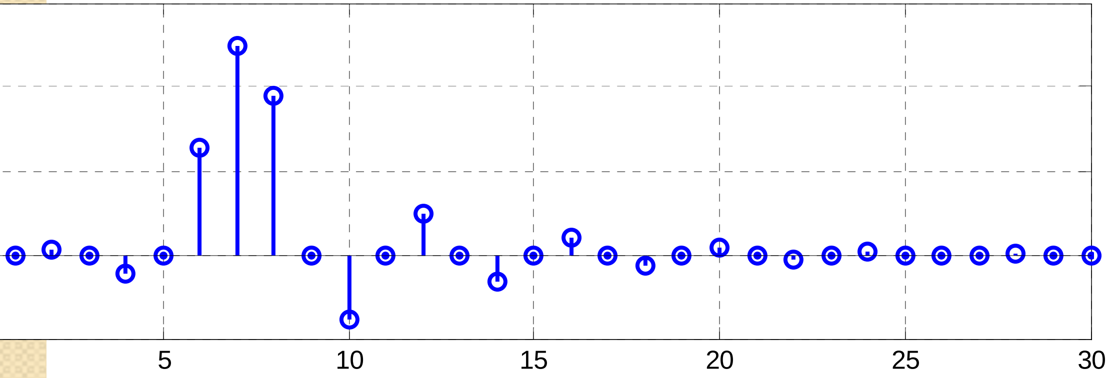
Filter

Channel
Select

……

DDS

Impulse Response, Two-Path, 4-Coefficient, Linear Phase IIR



$$N = 2.0 \cdot [1 + \frac{2}{2} + \frac{2}{4} + \frac{2}{8} + \cdots]$$

$$= 2.0 + 2.0 \cdot [1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots]$$

$$< 2.0 \cdot 3 = 6.0$$

Polyphase Partition of Band Pass Filter
1-Path to M-Path Transformation

Modulation Theorem of Z-Transform

$$G(Z) = \sum_{n=0}^{N-1} h(n)\, e^{j\theta_k n} Z^{-n} = \sum_{n=0}^{N-1} h(n)\,(e^{-j\theta_k} Z)^{-n} = H(e^{-j\theta_k} Z)$$

$$G(Z) = \sum_{r=0}^{M-1} \sum_{n=0}^{N-1} h(r+nM)\, e^{j\theta_k (r+nM)} Z^{-(r+nM)}$$

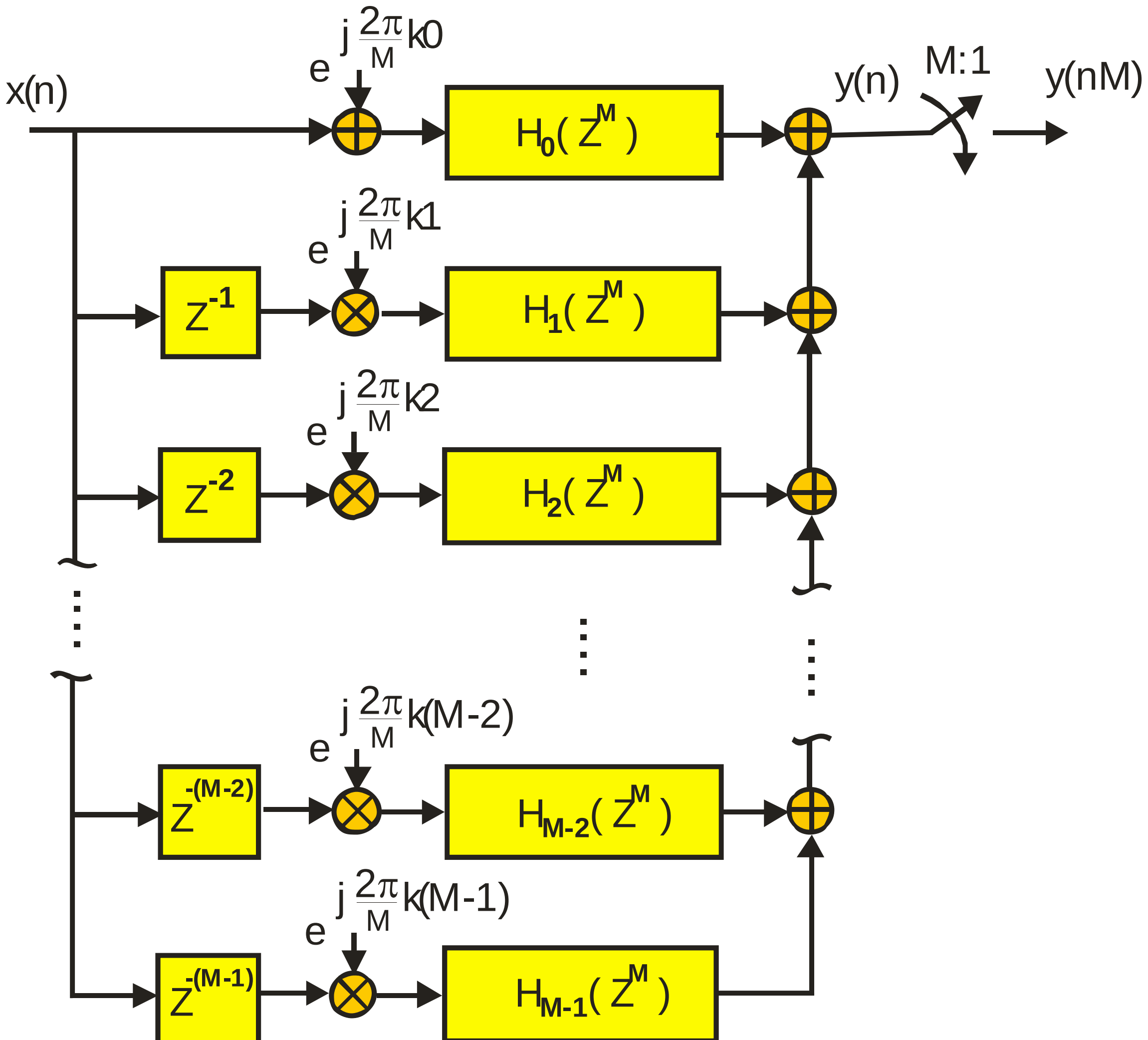
$$G(Z) = \sum_{r=0}^{M-1} e^{j\theta_k} Z^{-r} \sum_{n=0}^{N-1} h(r+nM) e^{j\theta_k nM} Z^{-nM}$$

$$\theta_k = k \cdot 2\pi$$

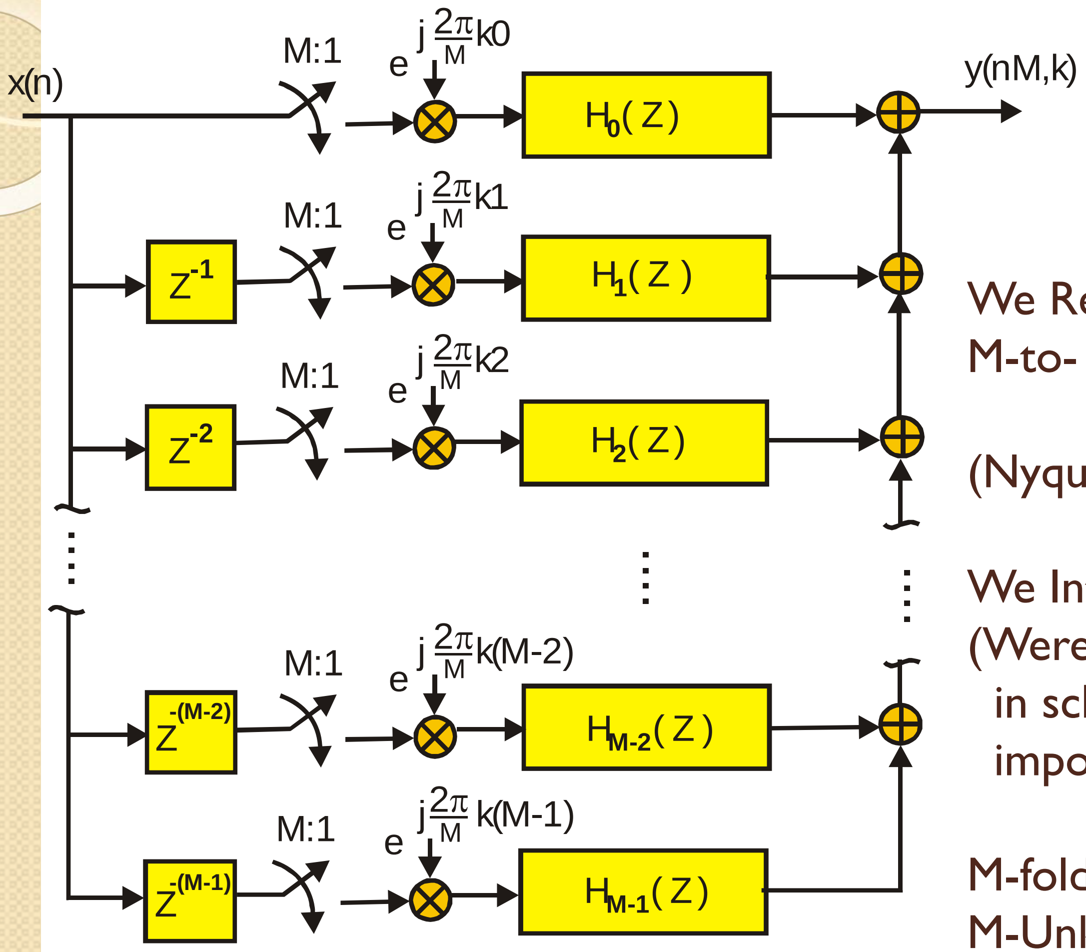
$$\theta_k = k \cdot \frac{2\pi}{M}$$

$$G(Z) = \sum_{r=0}^{M-1} e^{j\frac{2\pi}{M}k} Z^{-r} \sum_{n=0}^{N-1} h(r+nM) Z^{-nM}$$

Polyphase Band Pass Filter and M-to-1 Resampler



Apply Noble Identity to Polyphase Partition



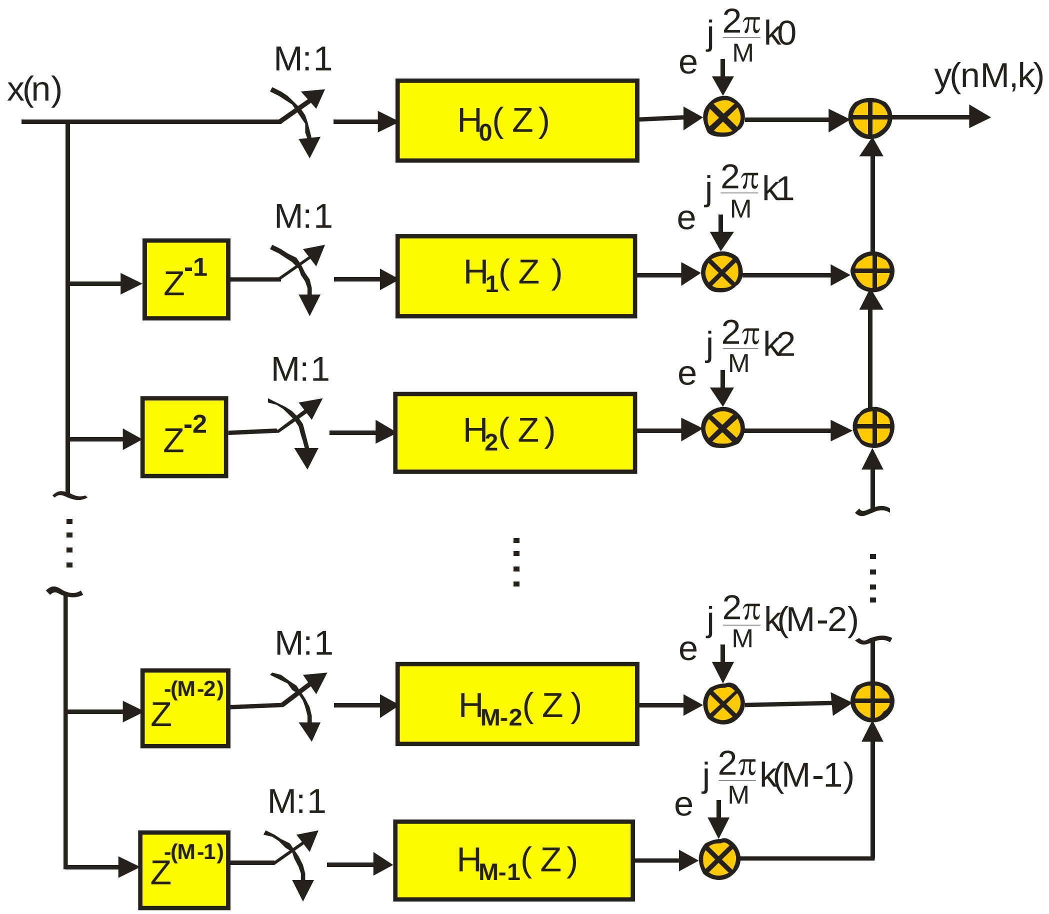
We Reduce Sample Rate
M-to-1 Prior to Reducing Bandwidth

(Nyquist is Raising His Eyebrows!)

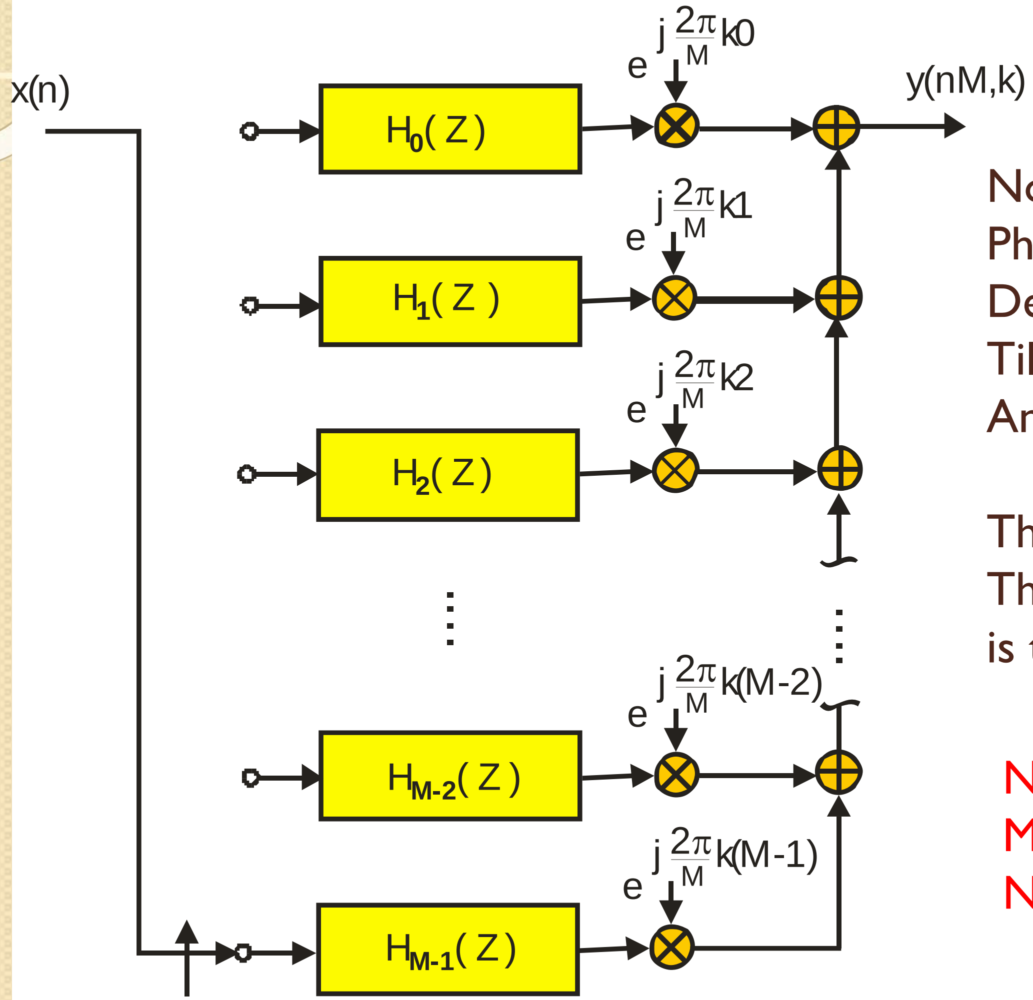
We Intentionally Alias the Spectrum.
(Were you Paying Attention
in school when they discussed the
importance of anti-aliasing filters?)

M-fold Aliasing!
M-Unknowns!
M-Paths supply M-Equations
We can the separate Aliases!

Move Phase Spinners to Output of Polyphase Filter Paths



Polyphase Partition with Commutator Replacing the "r" Delays in the "r-th" Path

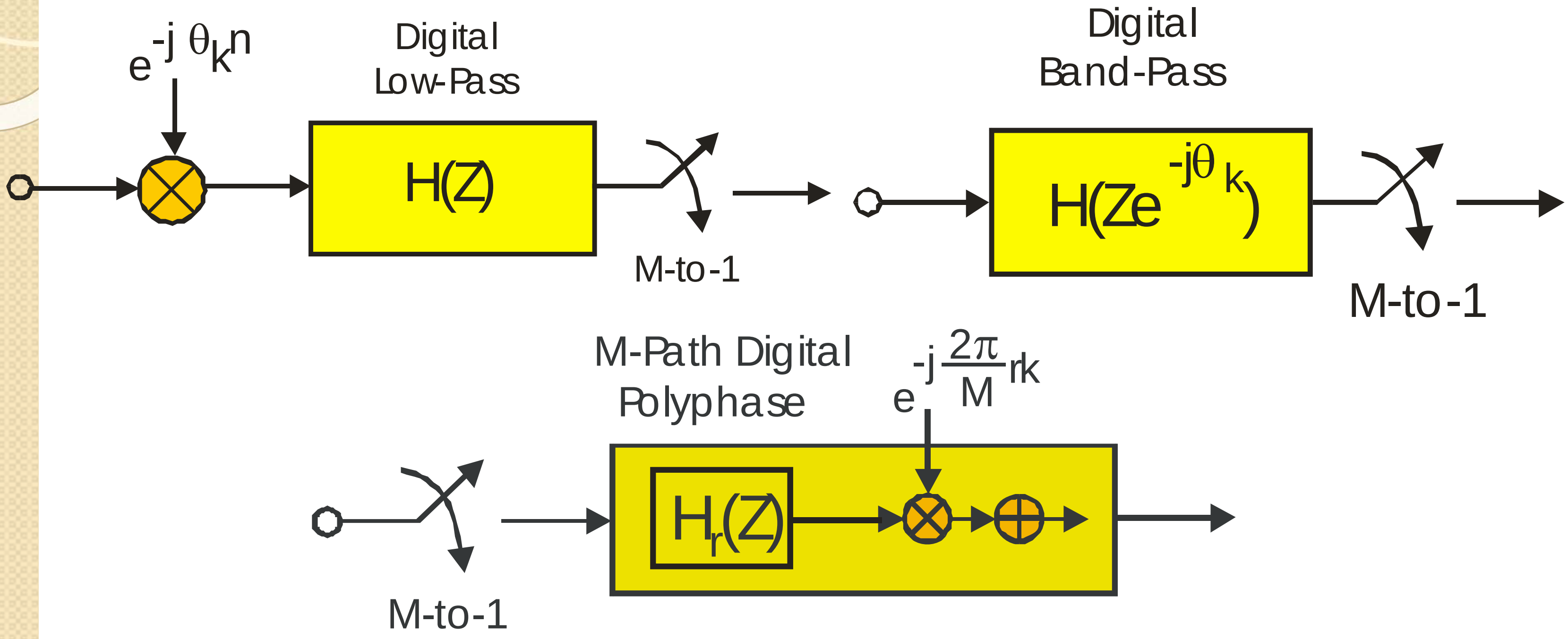


Note: We don't assign Phase Spinners to Select Desired Center Frequency Till after Down Sampling And Path Processing

This Means that The Processing for every Channel is the same till the Phase Spinner

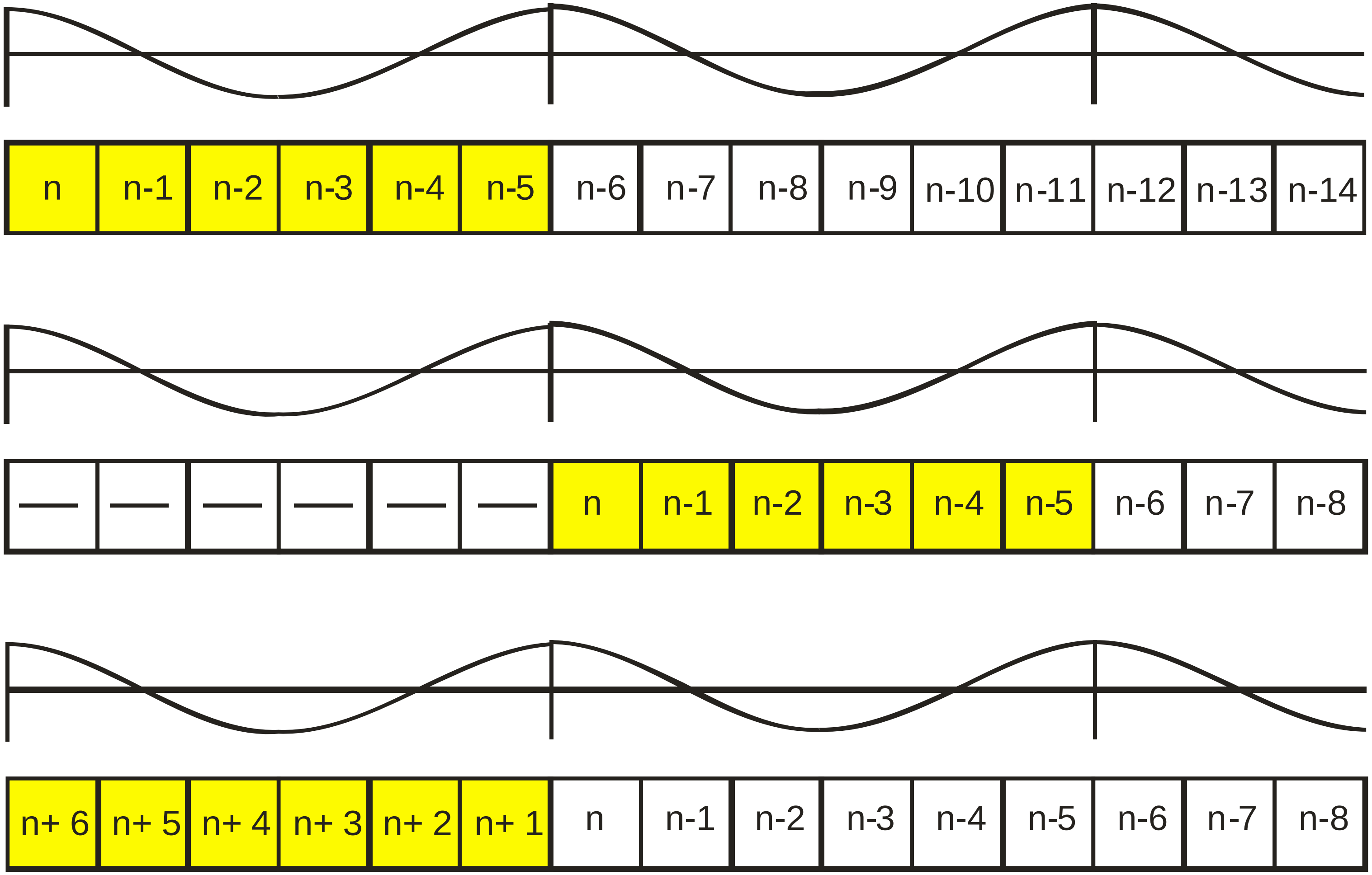
No longer LTI, Filter now has M-Different Impulse Responses! Now LTV or PTV Filter.

Armstrong to Tuned RF with Alias Down Conversion to Polyphase Receiver



Rather than selecting center frequency at input and reduce sample rate at output, we reverse the order, reduce sample rate at input and select center frequency at output. We perform arithmetic operations at low output rate rather than at high input rate!

Down Sample 6-to-1

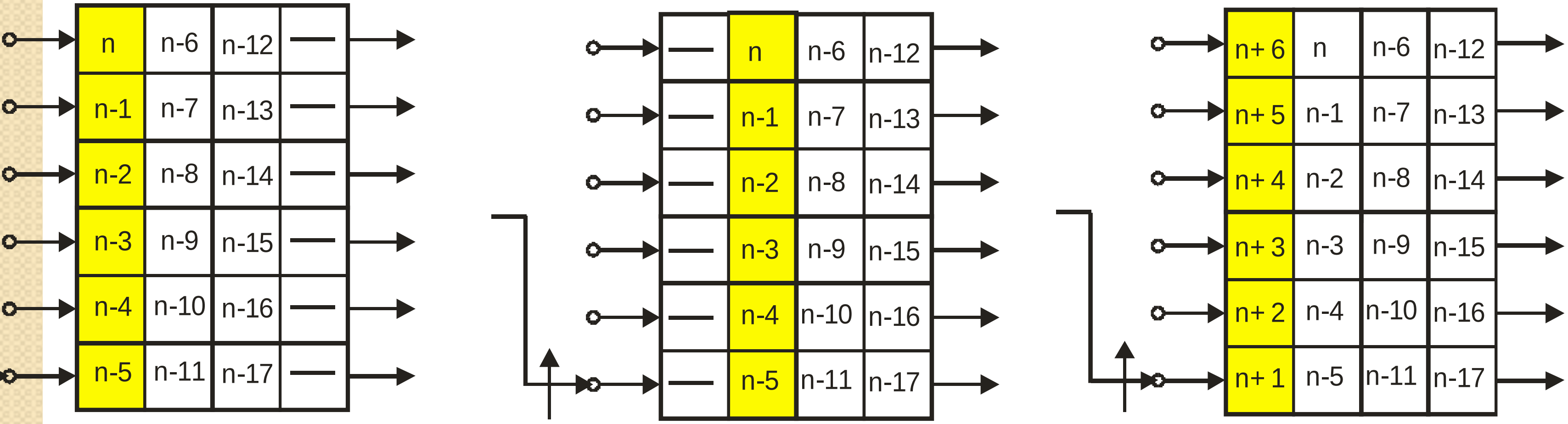


n	n-1	n-2	n-3	n-4	n-5	n-6	n-7	n-8	n-9	n-10	n-11	n-12	n-13	n-14

—	—	—	—	—	—	n	n-1	n-2	n-3	n-4	n-5	n-6	n-7	n-8

n+ 6	n+ 5	n+ 4	n+ 3	n+ 2	n+ 1	n	n-1	n-2	n-3	n-4	n-5	n-6	n-7	n-8

Polyphase Partition 1-D filter becomes 2-D M-Path Filter

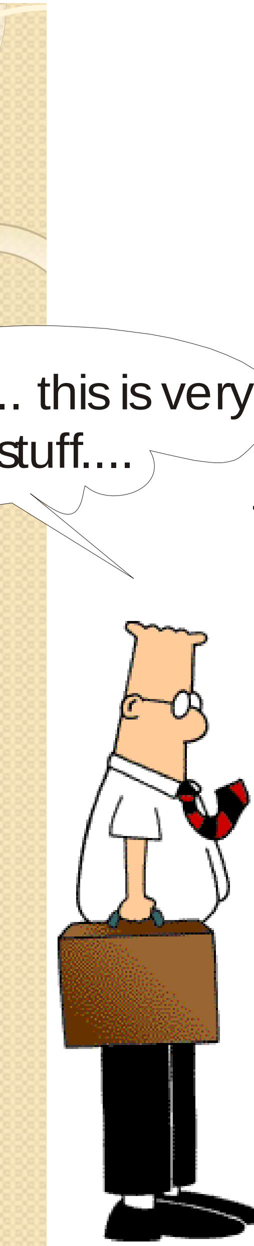


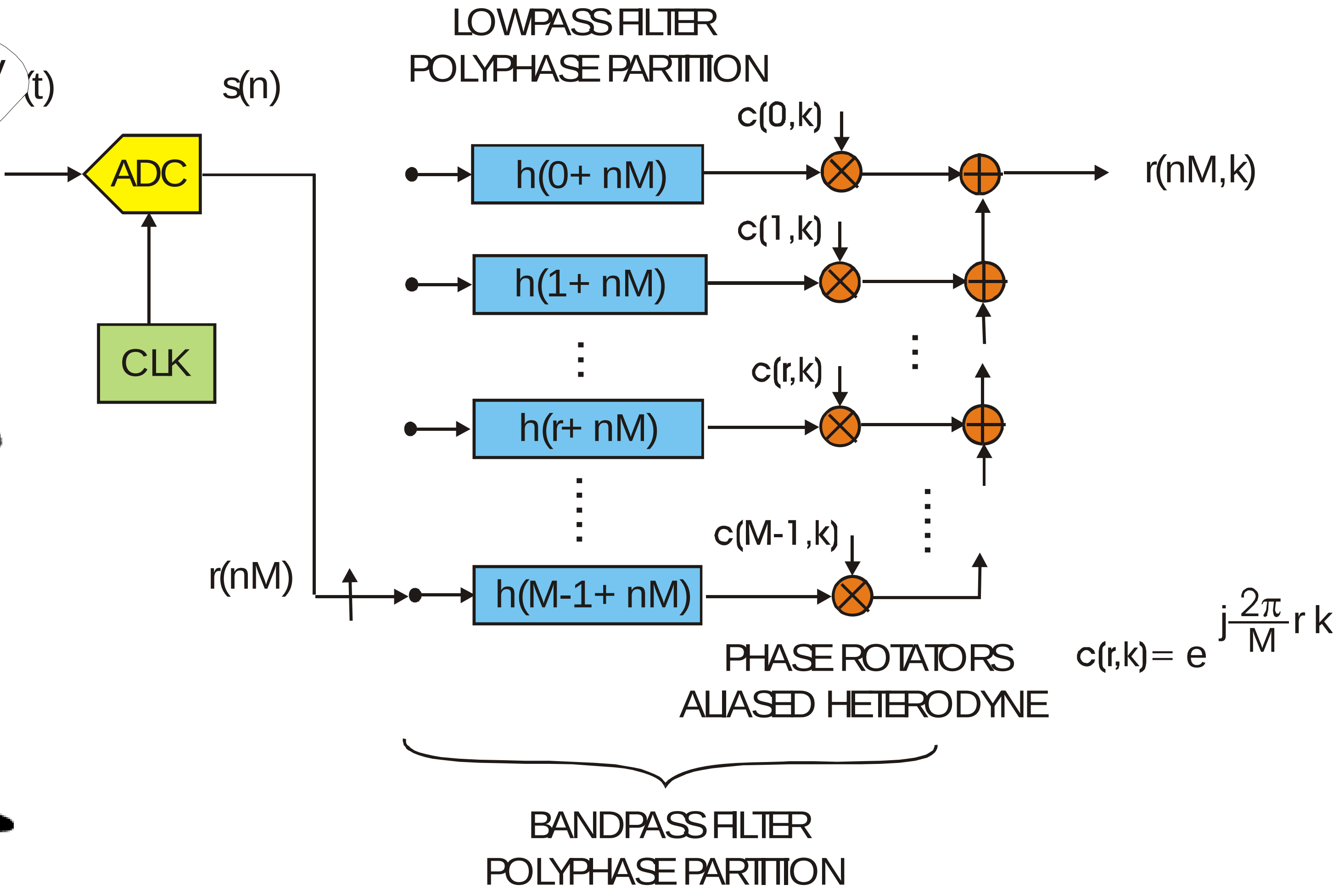
n	n-6	n-12	—
n-1	n-7	n-13	—
n-2	n-8	n-14	—
n-3	n-9	n-15	—
n-4	n-10	n-16	—
n-5	n-11	n-17	—

—	n	n-6	n-12
—	n-1	n-7	n-13
—	n-2	n-8	n-14
—	n-3	n-9	n-15
—	n-4	n-10	n-16
—	n-5	n-11	n-17

n+ 6	n	n-6	n-12
n+ 5	n-1	n-7	n-13
n+ 4	n-2	n-8	n-14
n+ 3	n-3	n-9	n-15
n+ 2	n-4	n-10	n-16
n+ 1	n-5	n-11	n-17

Reorder Filter and Resample




LOWPASS FILTER
POLYPHASE PARTITION

(t)

s(n)

ADC

CLK

r(nM)

c(0,k)

h(0+ nM)

r(nM,k)

c(1,k)

h(1+ nM)

c(r,k)

h(r+ nM)

c(M-1,k)

h(M-1+ nM)

PHASE ROTATORS
ALIASED HETERODYNE

$$c(r,k) = e^{j\frac{2\pi}{M} r\, k}$$

BANDPASS FILTER
POLYPHASE PARTITION

Phase and Gain Response

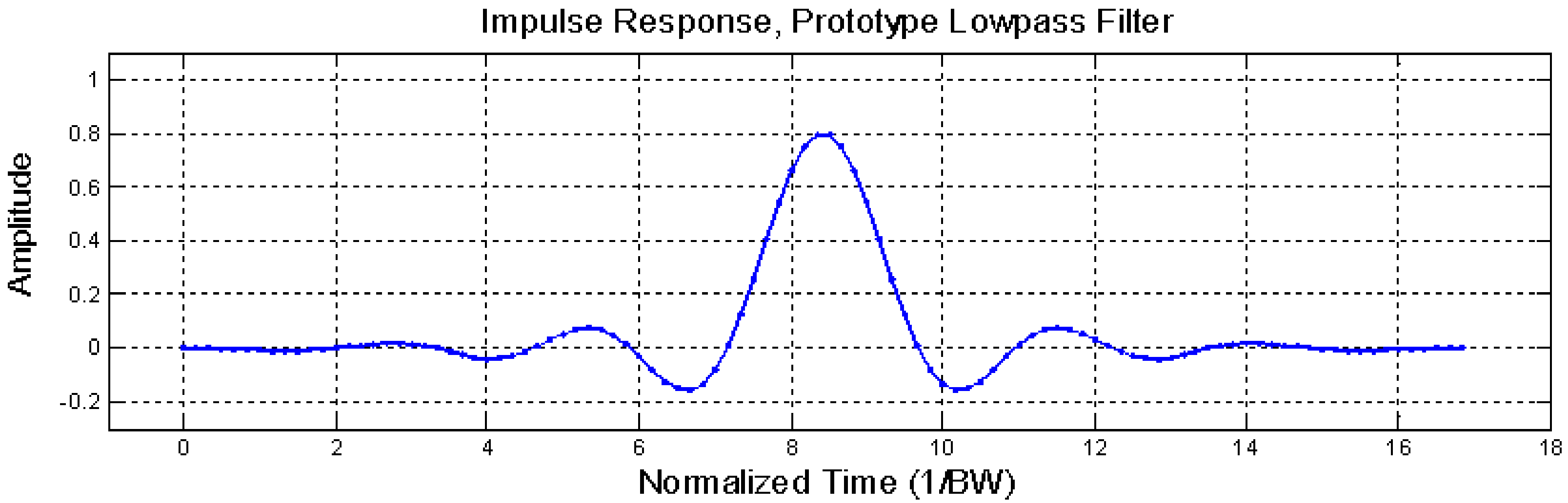
(3-Versions of Filter)

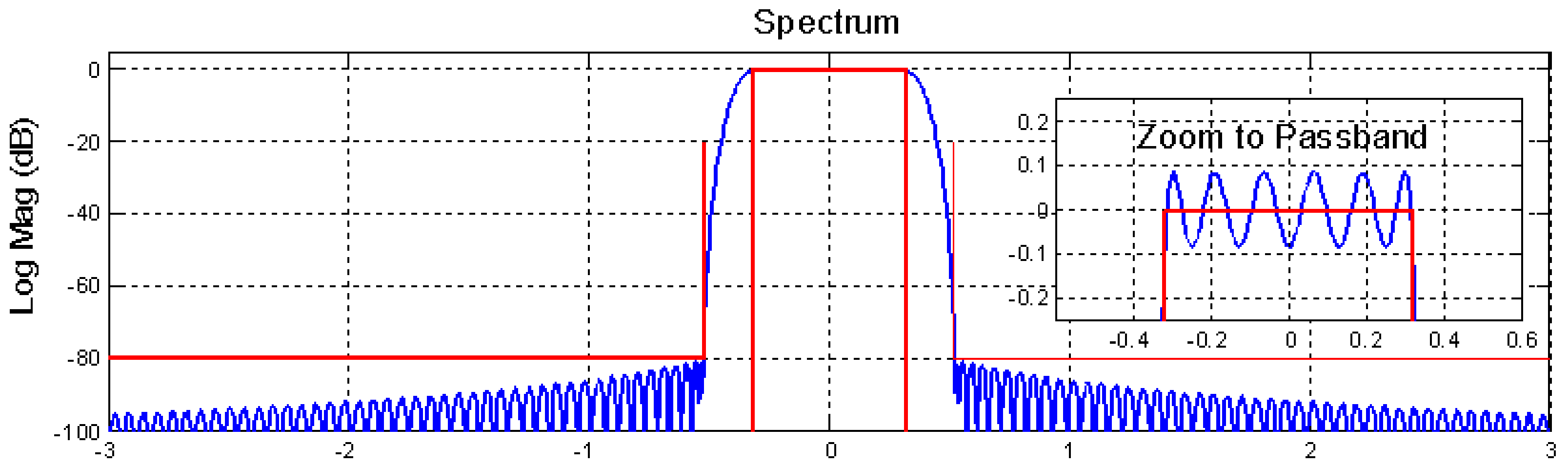
Prototype Filter,

Polyphase Filter Prior to Resampling,

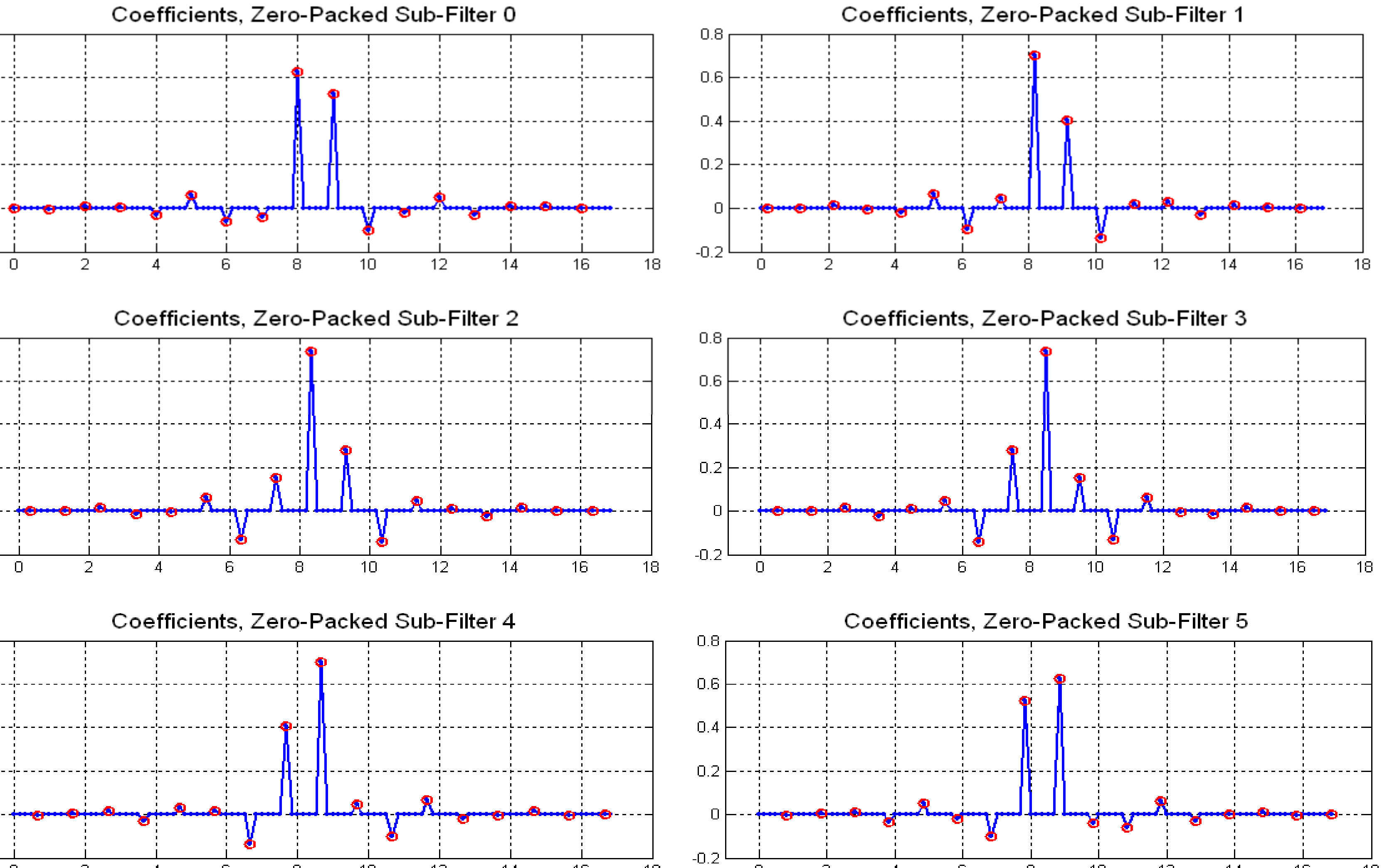
Polyphase Filter after Resampling

Impulse Response and Frequency Response of Prototype Low Pass FIR Filter





Impulse Response of 6-Path Polyphase Partition Prior to 6-to-1 Resampling


Coefficients, Zero-Packed Sub-Filter 0

Coefficients, Zero-Packed Sub-Filter 1

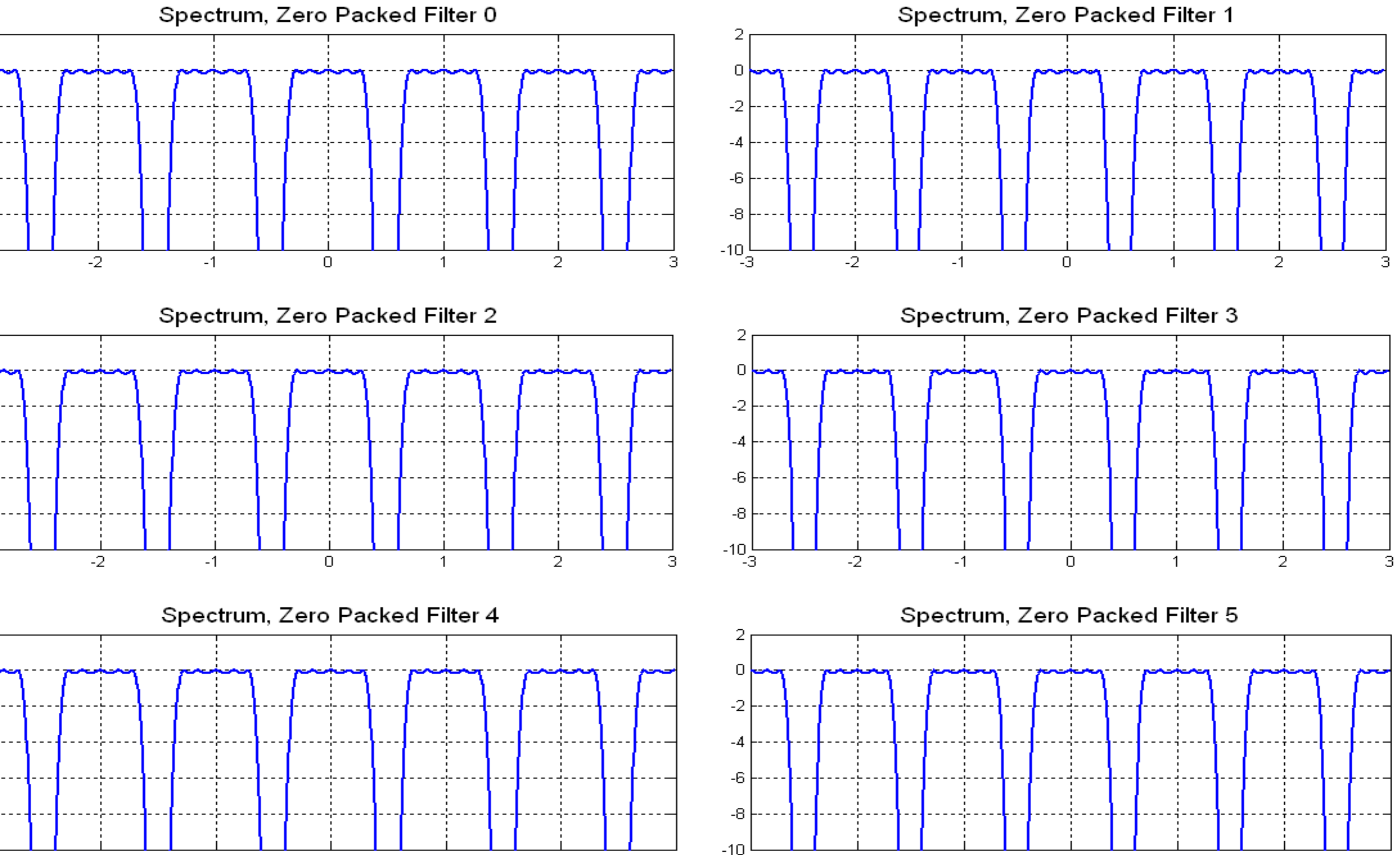
Coefficients, Zero-Packed Sub-Filter 2

Coefficients, Zero-Packed Sub-Filter 3

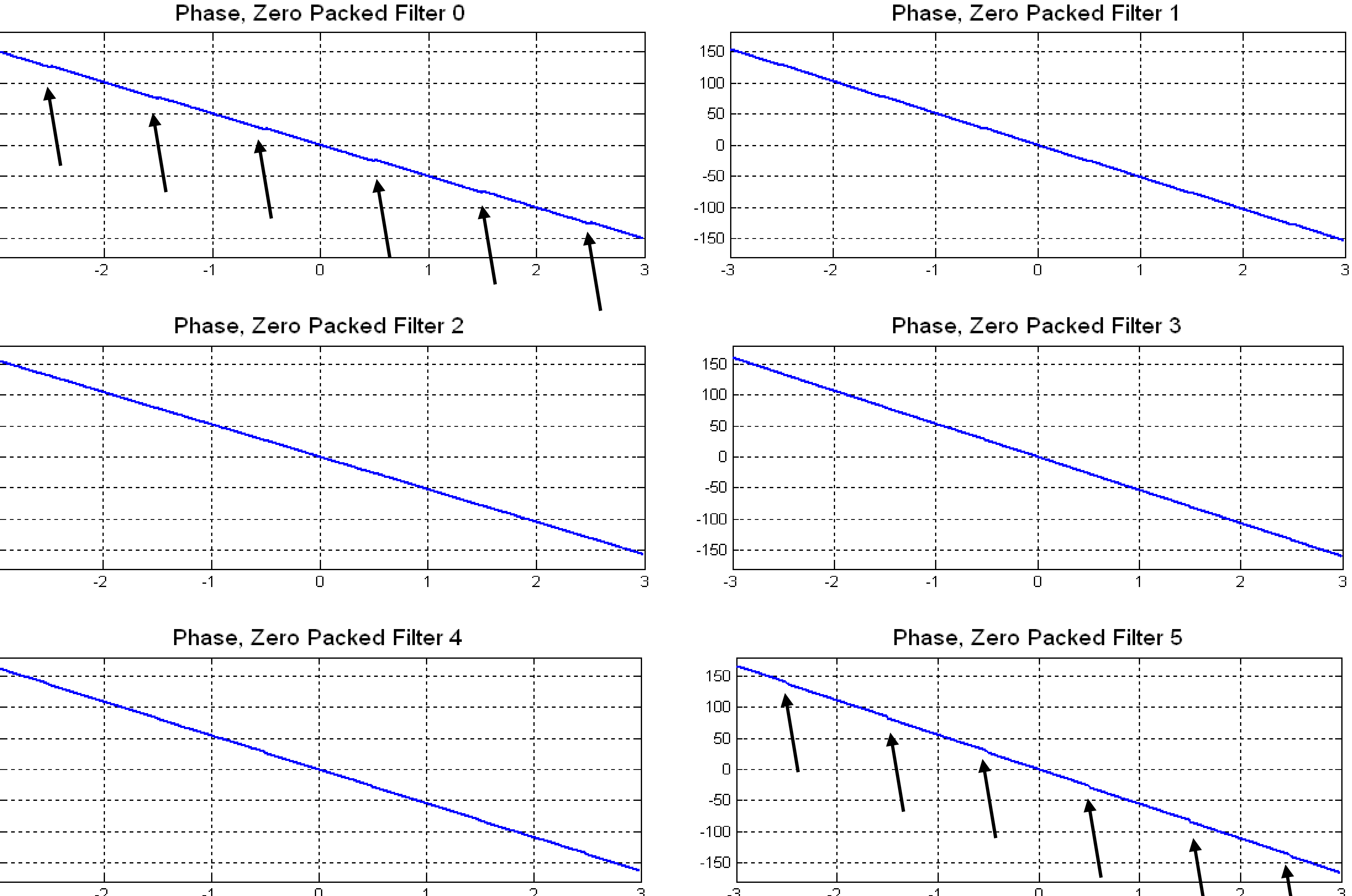
Coefficients, Zero-Packed Sub-Filter 4

Coefficients, Zero-Packed Sub-Filter 5

Frequency Response of 6-Path Polyphase Partition Prior to 6-to-1 Resampling



Phase Response of 6-Path Polyphase Partition Prior to 6-to-1 Resampling


Phase, Zero Packed Filter 0

Phase, Zero Packed Filter 1

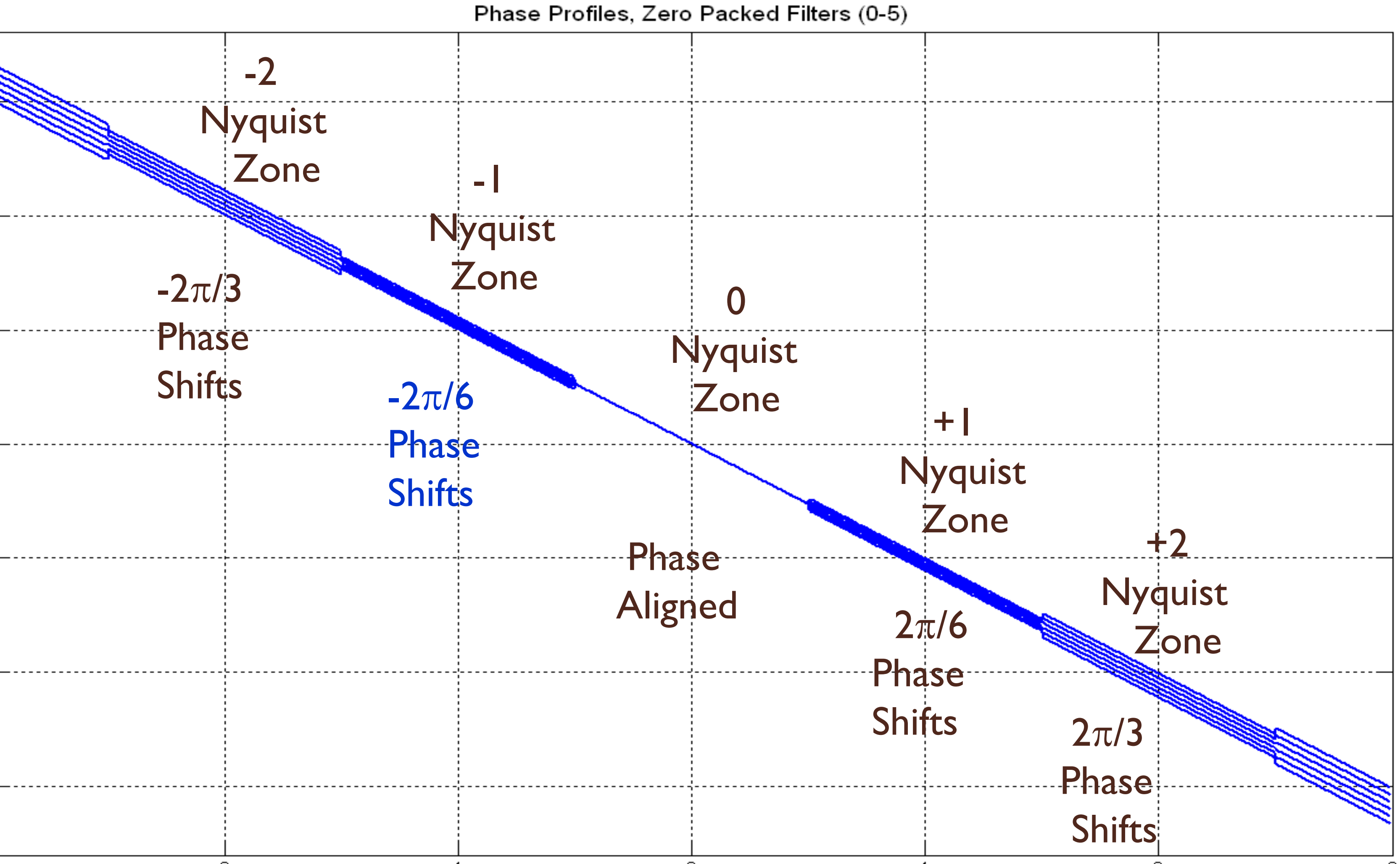
Phase, Zero Packed Filter 2

Phase, Zero Packed Filter 3

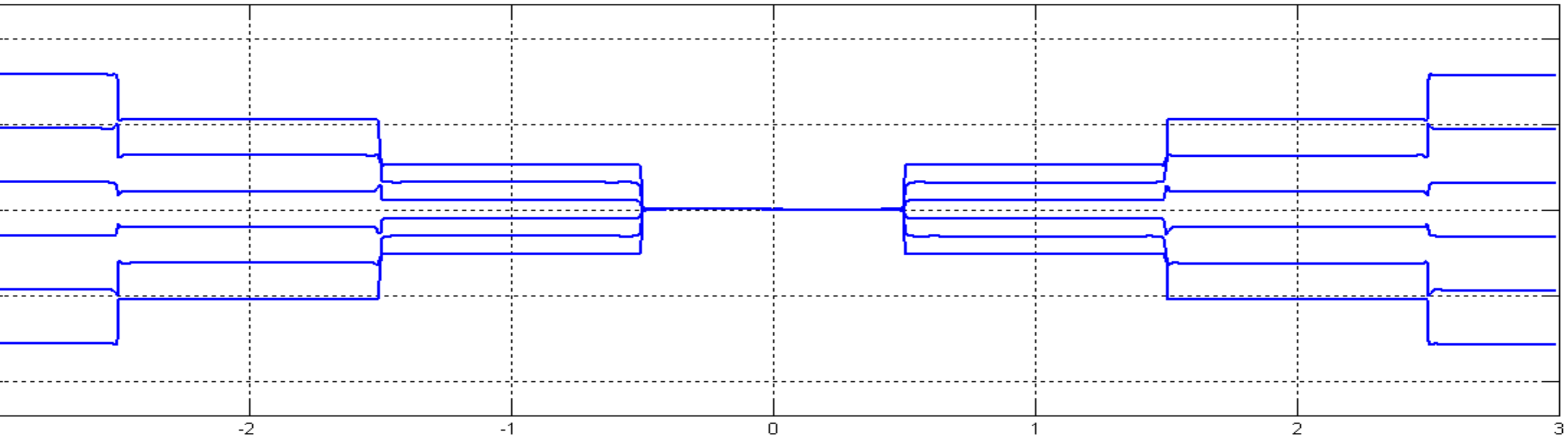
Phase, Zero Packed Filter 4

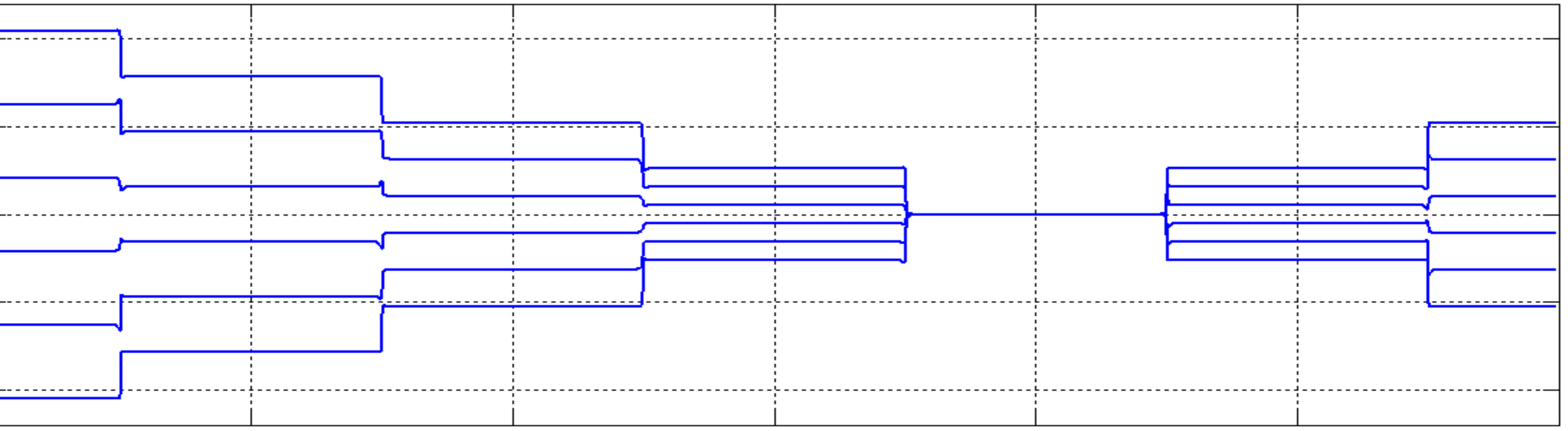
Phase, Zero Packed Filter 5

Overlay Phase Response of 6-Path Polyphase Partition Prior to 6-to-1 Resampling

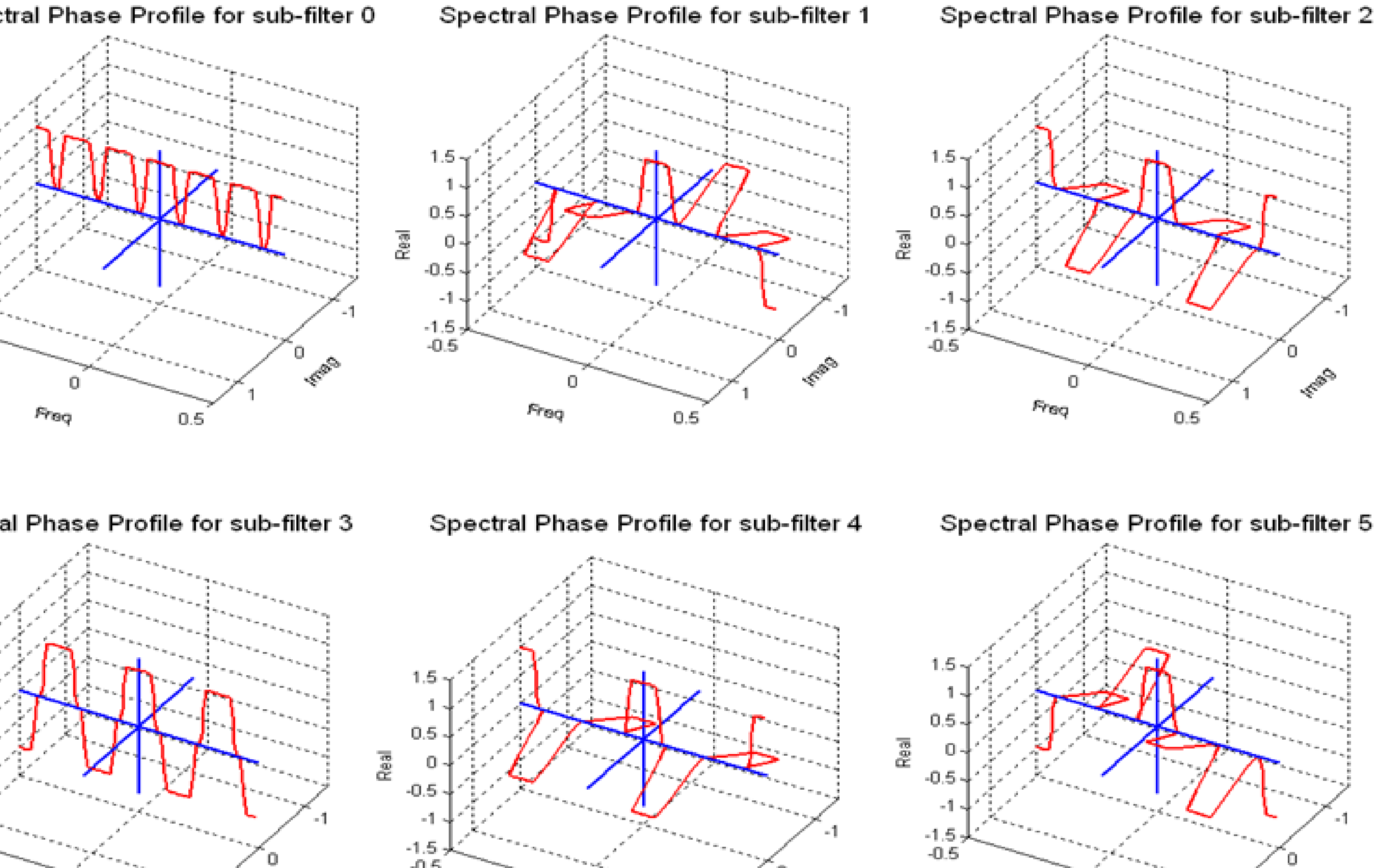


De-Trended Overlay Phase Response:
6-Path Partition Prior to 6-to-1 Resampling

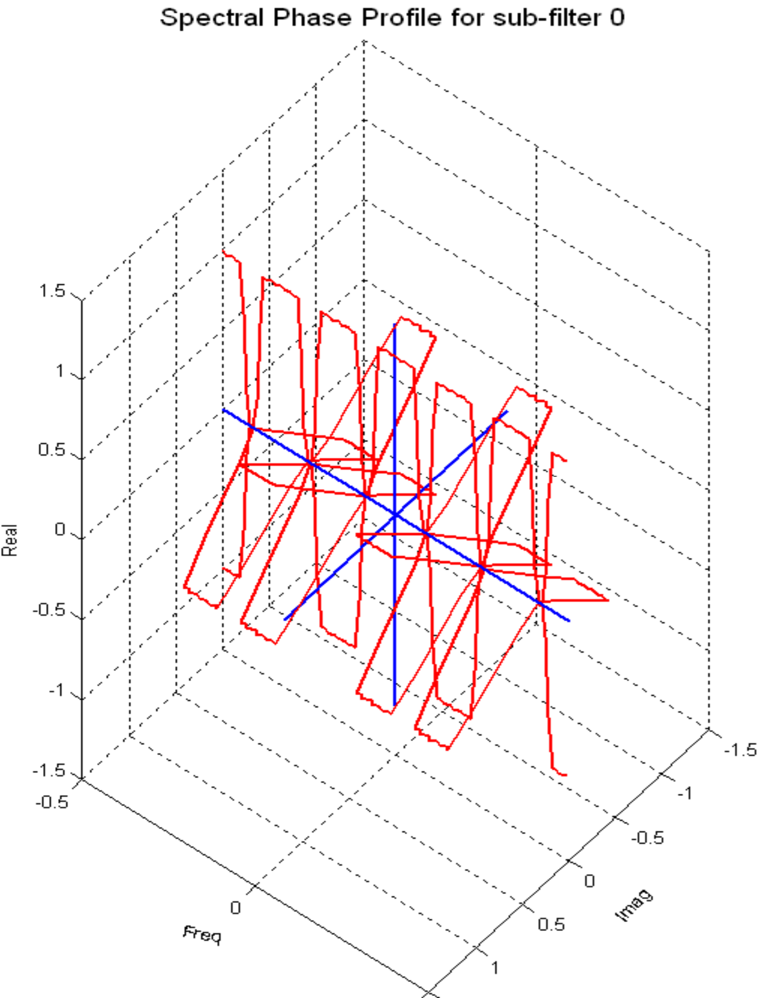

Phase Profiles Non-Causal Sub-Filters (0-5)

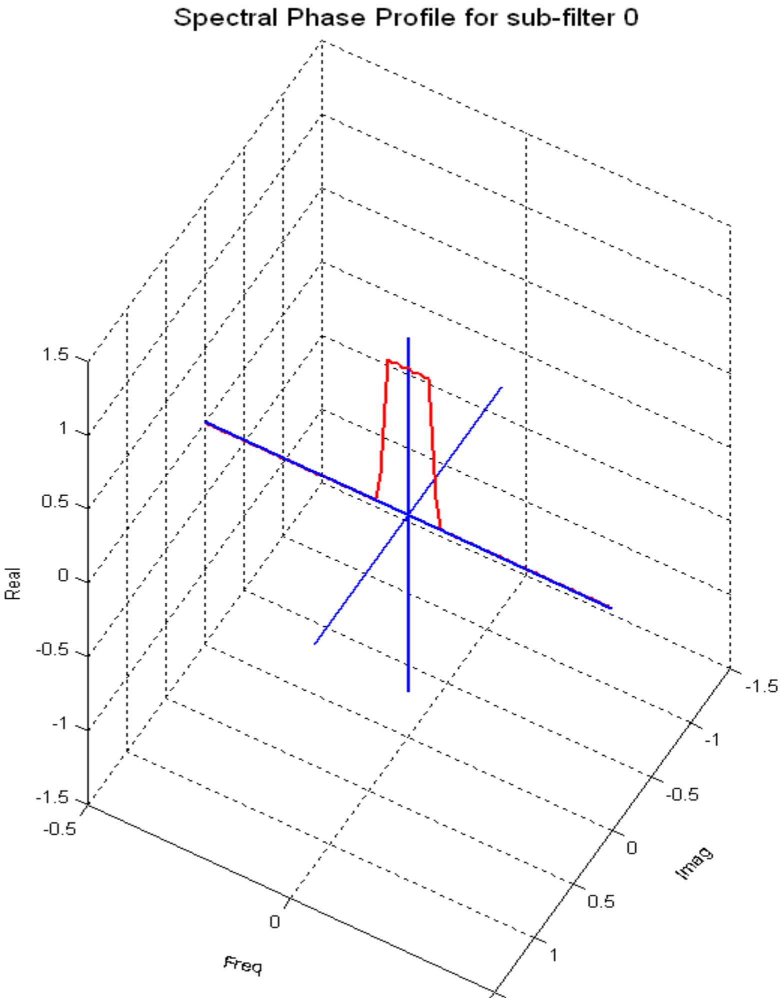

Phase Profiles Non-Causal Phase Rotated sub-filters (0-5)

3-D Paddle-Wheel Phase Profiles, 6-Path Partition Prior to 6-to-1 Resampling

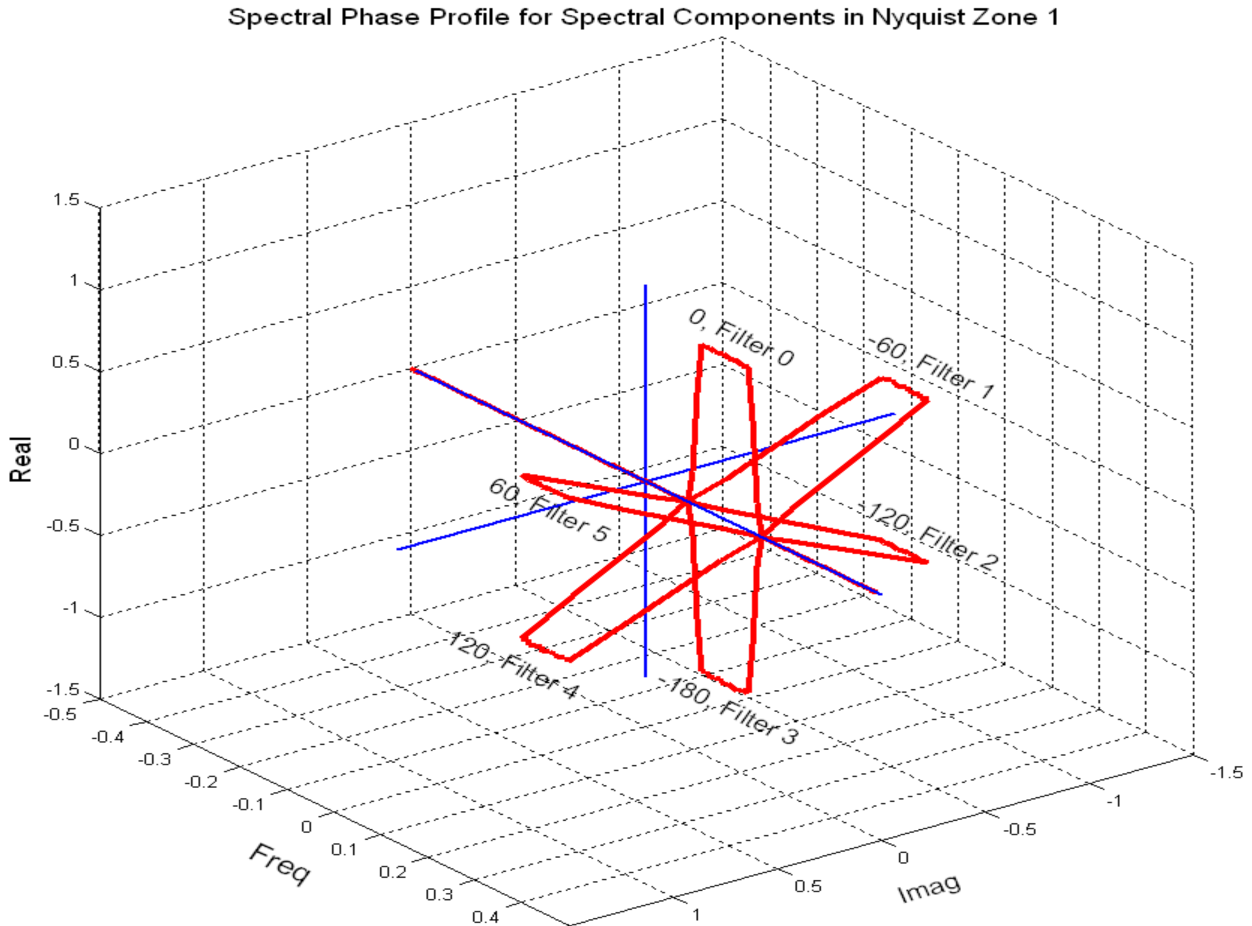


Overlay 3-D Paddle-Wheel Phase Profiles, 6-Path Partition Prior to 6-to-1 Resampling

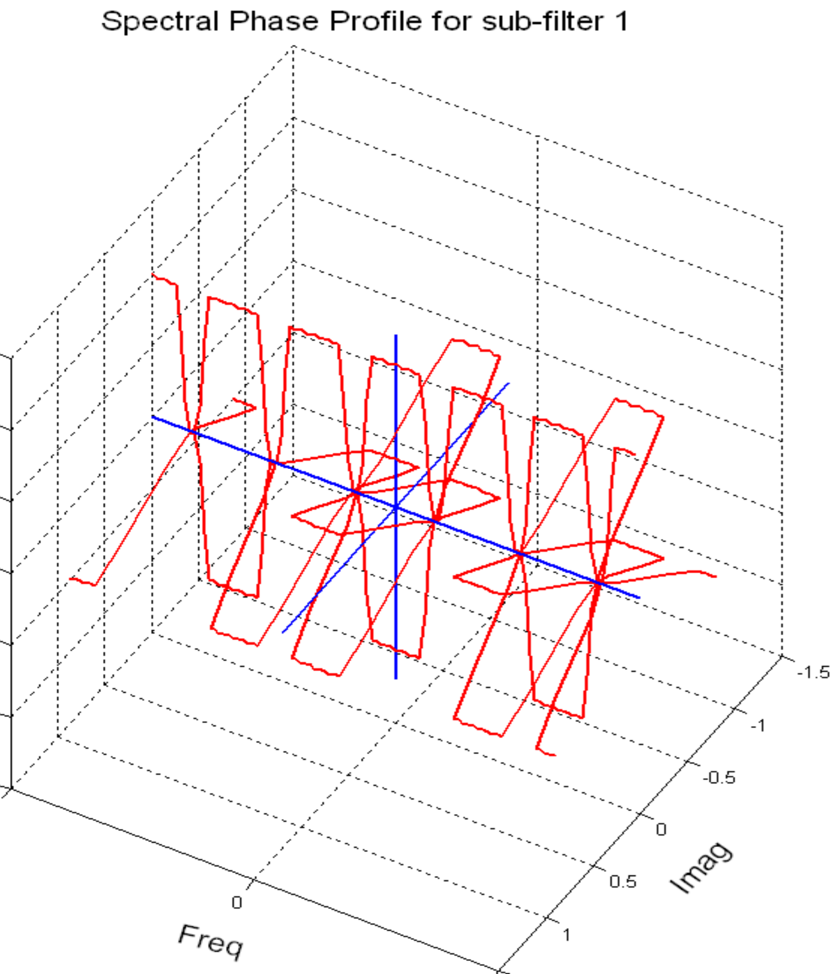

Spectral Phase Profile for sub-filter 0

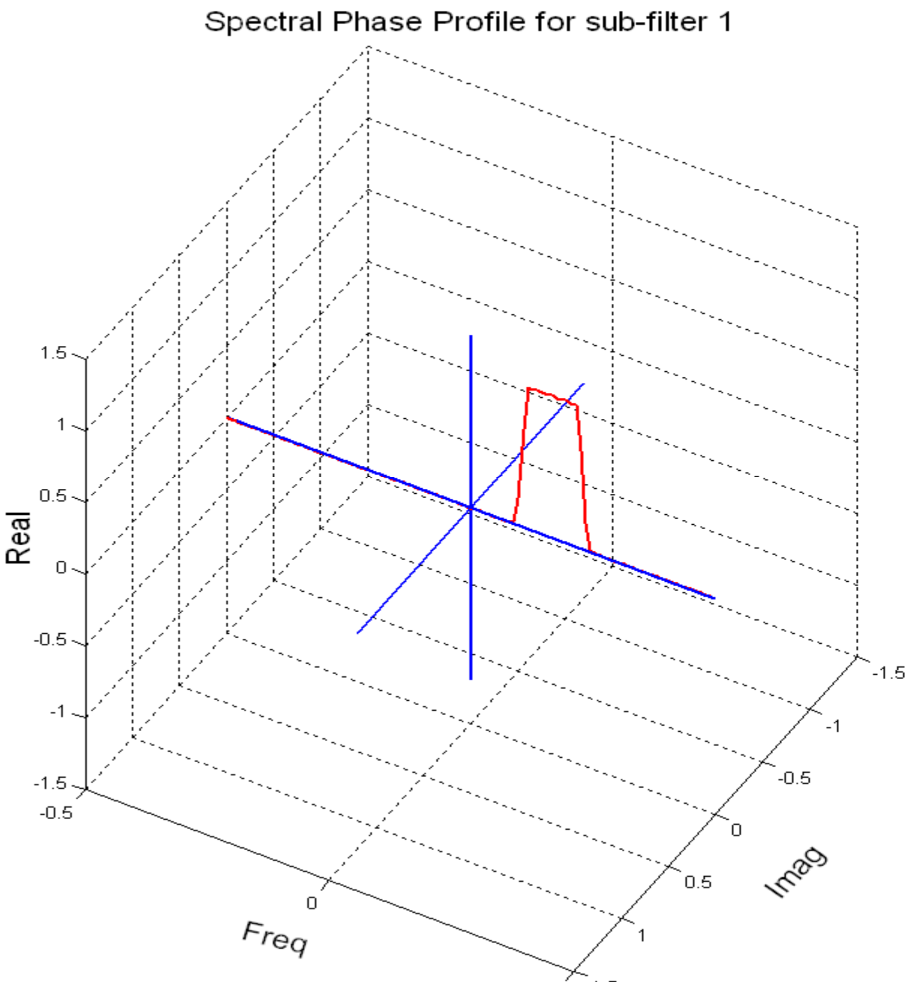

Spectral Phase Profile for sub-filter 0

Overlay 3-D Paddle-Wheel Phase Profiles, Showing Phase Shifts in +1 Nyquist Zone



Overlay 3-D Paddle-Wheel Phase Profiles, Phase Shifted to Align Phases in +1 Nyquist Zone

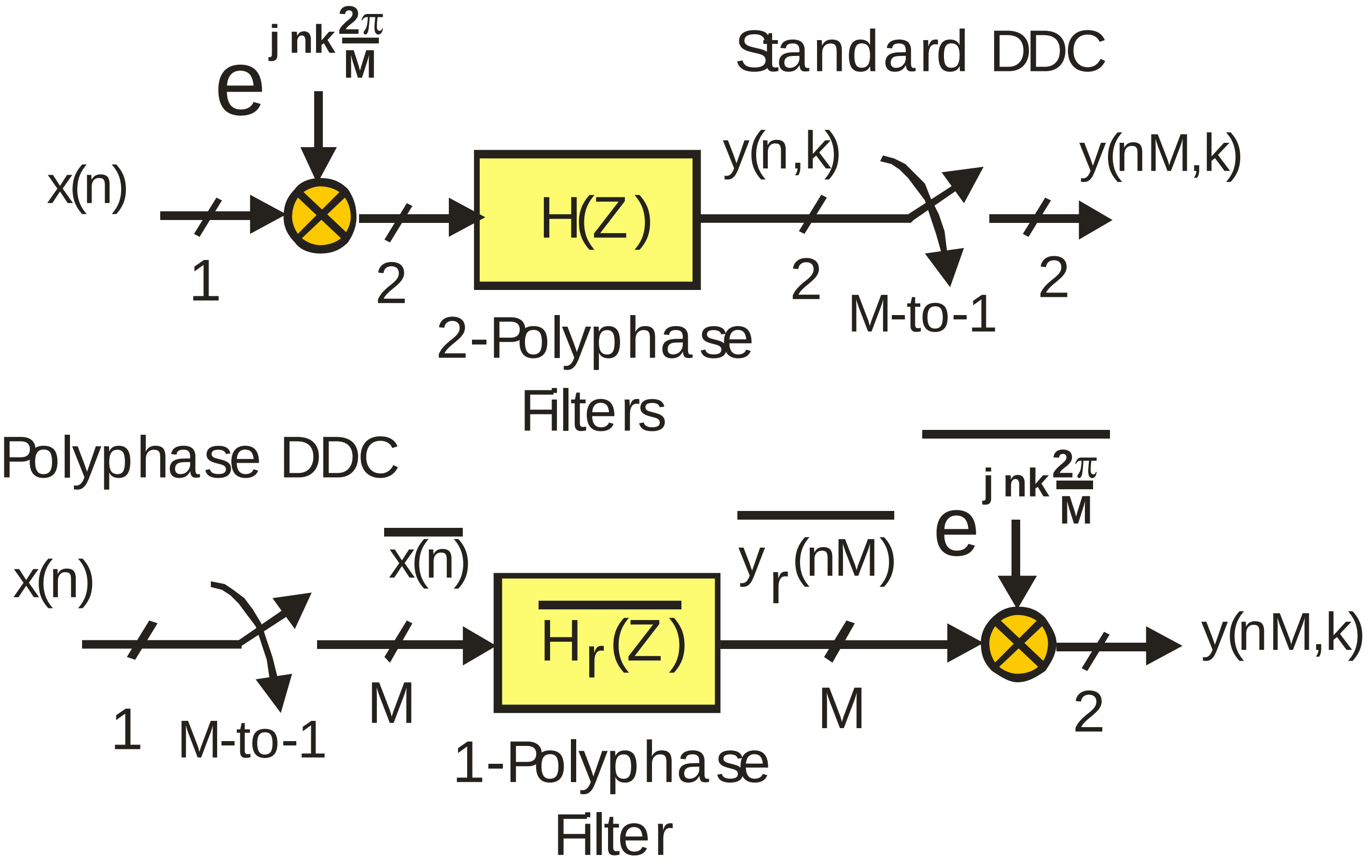

Spectral Phase Profile for sub-filter 1


Spectral Phase Profile for sub-filter 1

MATLAB DEMO-1

PolyChanDemo

Single Channel Armstrong and Multirate Aliased Polyphase Receiver



Standard DDC

$e^{jnk\frac{2\pi}{M}}$

x(n)

1

2

H(Z)

2-Polyphase Filters

y(n,k)

2

M-to-1

y(nM,k)

2

Polyphase DDC

x(n)

1

M-to-1

$\overline{x(n)}$

M

$\overline{H_r(Z)}$

1-Polyphase Filter

$\overline{y_r(nM)}$

M

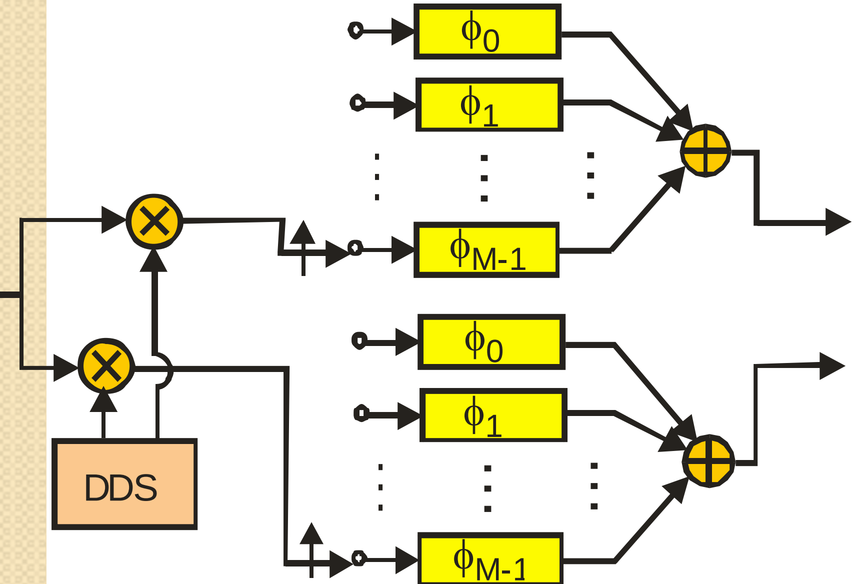
$\overline{e^{jnk\frac{2\pi}{M}}}$

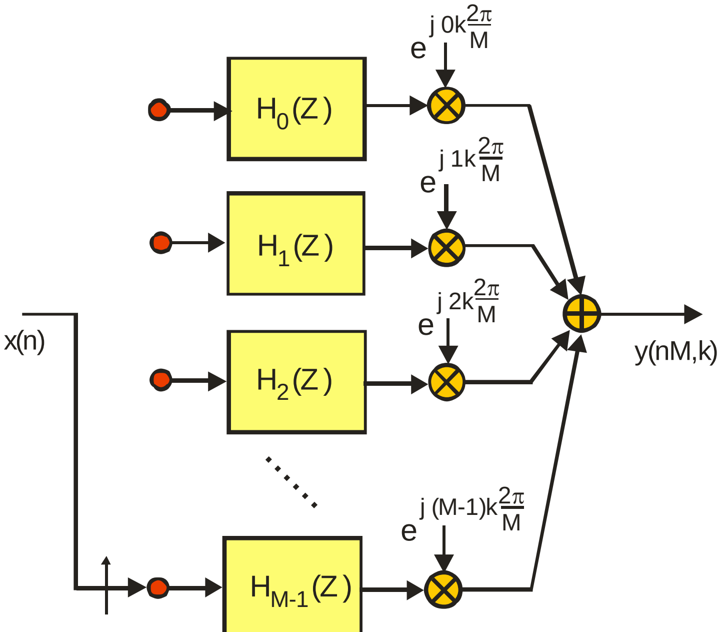
y(nM,k)

2

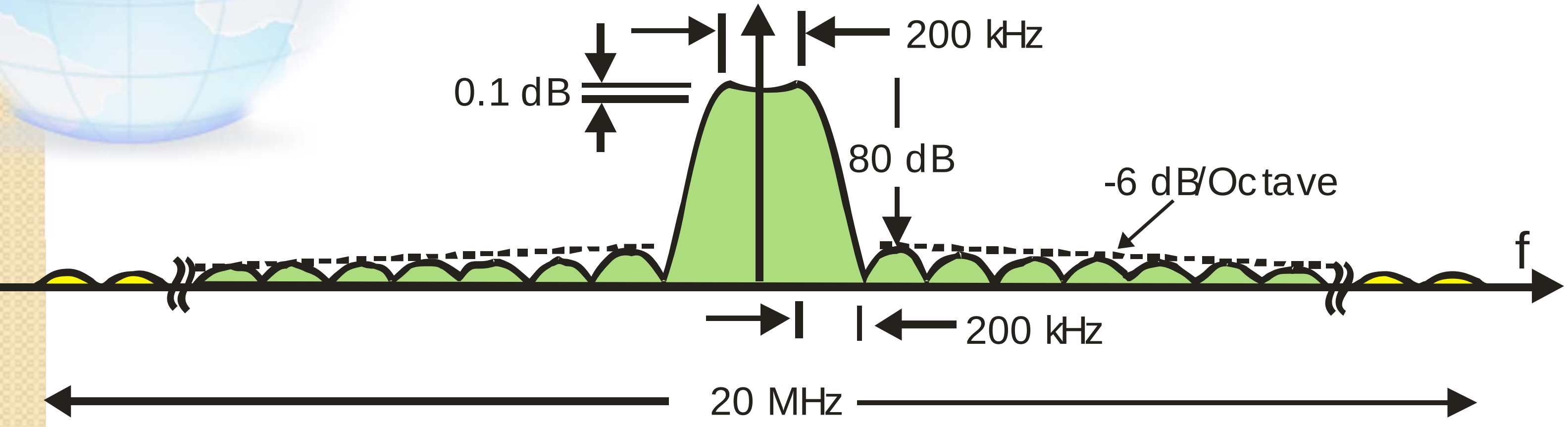
ide Input Down Conversion to Output of Filter Where it anishes Due to Down Sampling. Rotators in Filter Factor Out and are Applied to Path Outputs Rather than to Coefficients.

Advantage: Real sequence is made complex at output of Filter Rather than at Input to Filter





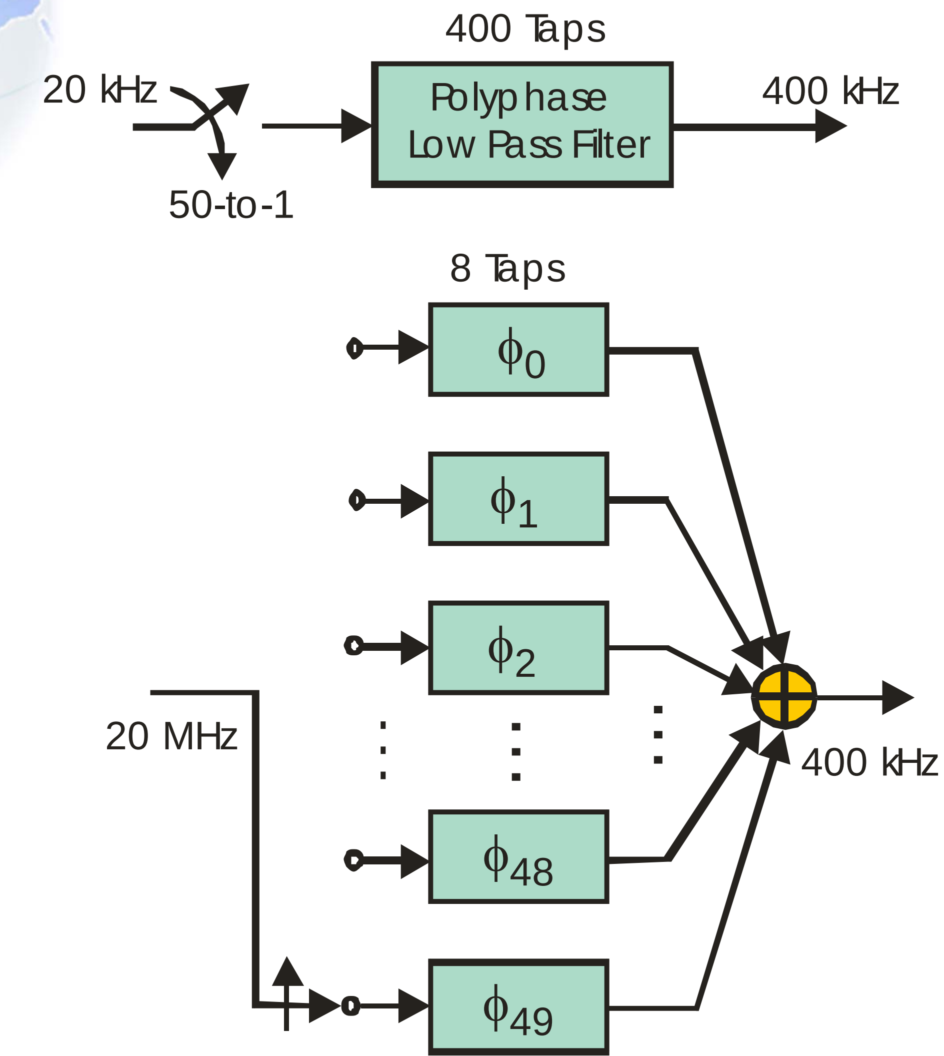
Bad Mismatch: Sample Rate Large Compared to Transition Bandwidth



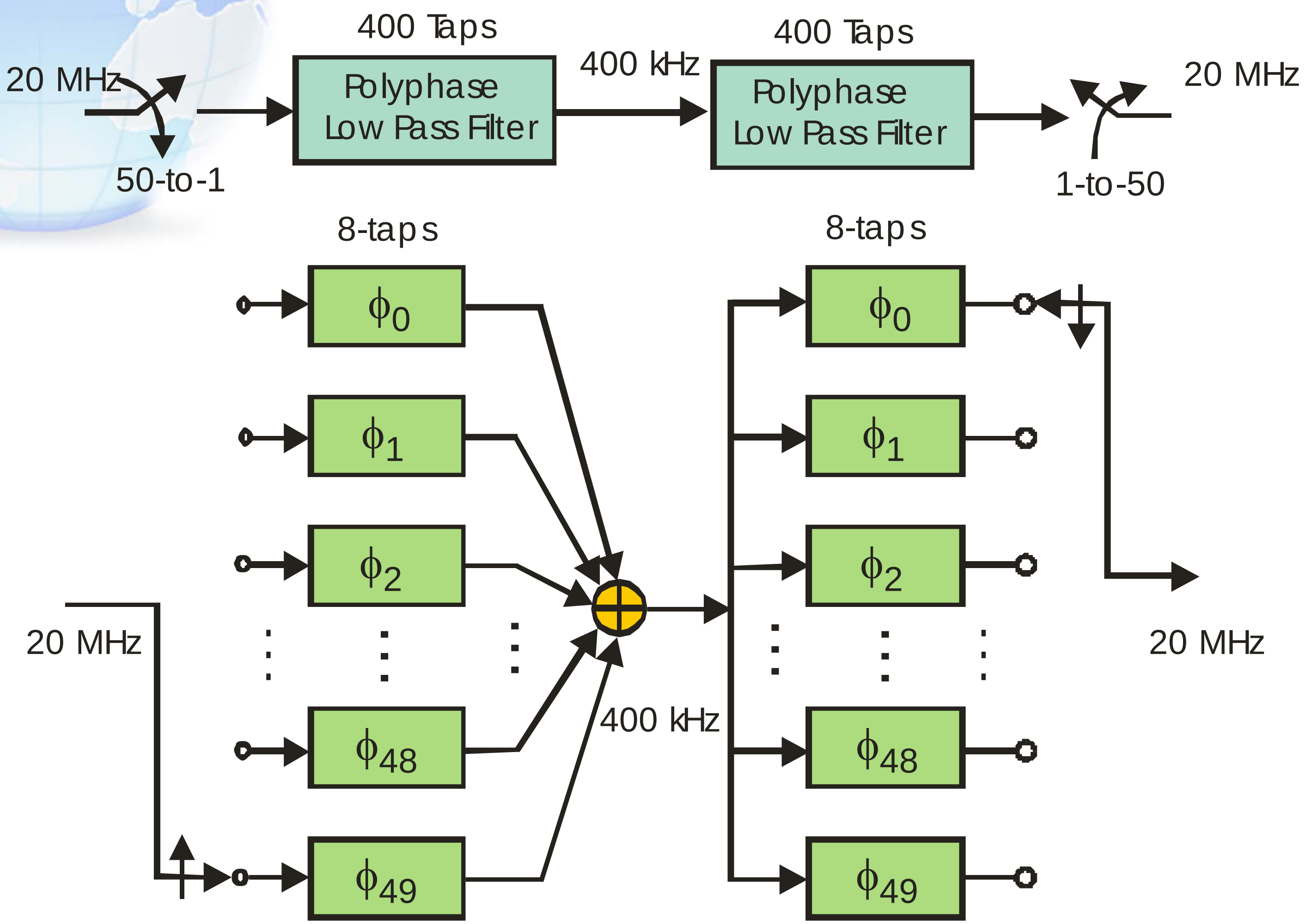
Nyquist Rate for Filter is

200 kHz + 200 kHz = 400 kHz or fs/50

Polyphase Partition of Low-Pass Filter



Cascade Polyphase Filter
Down-Sampling and Up-Sampling



400 Taps

Polyphase
Low Pass Filter

400 kHz

400 Taps

Polyphase
Low Pass Filter

20 MHz

50-to-1

20 MHz

1-to-50

8-taps

8-taps

ϕ_0

ϕ_1

ϕ_2

ϕ_{48}

ϕ_{49}

ϕ_0

ϕ_1

ϕ_2

ϕ_{48}

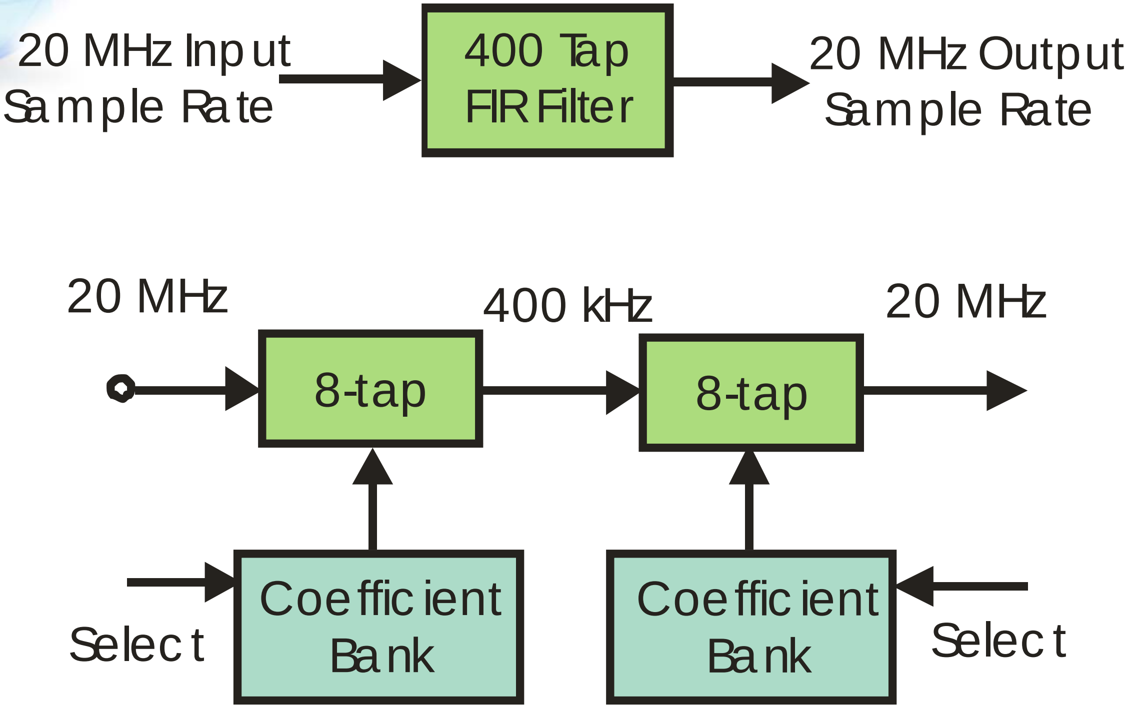
ϕ_{49}

20 MHz

400 kHz

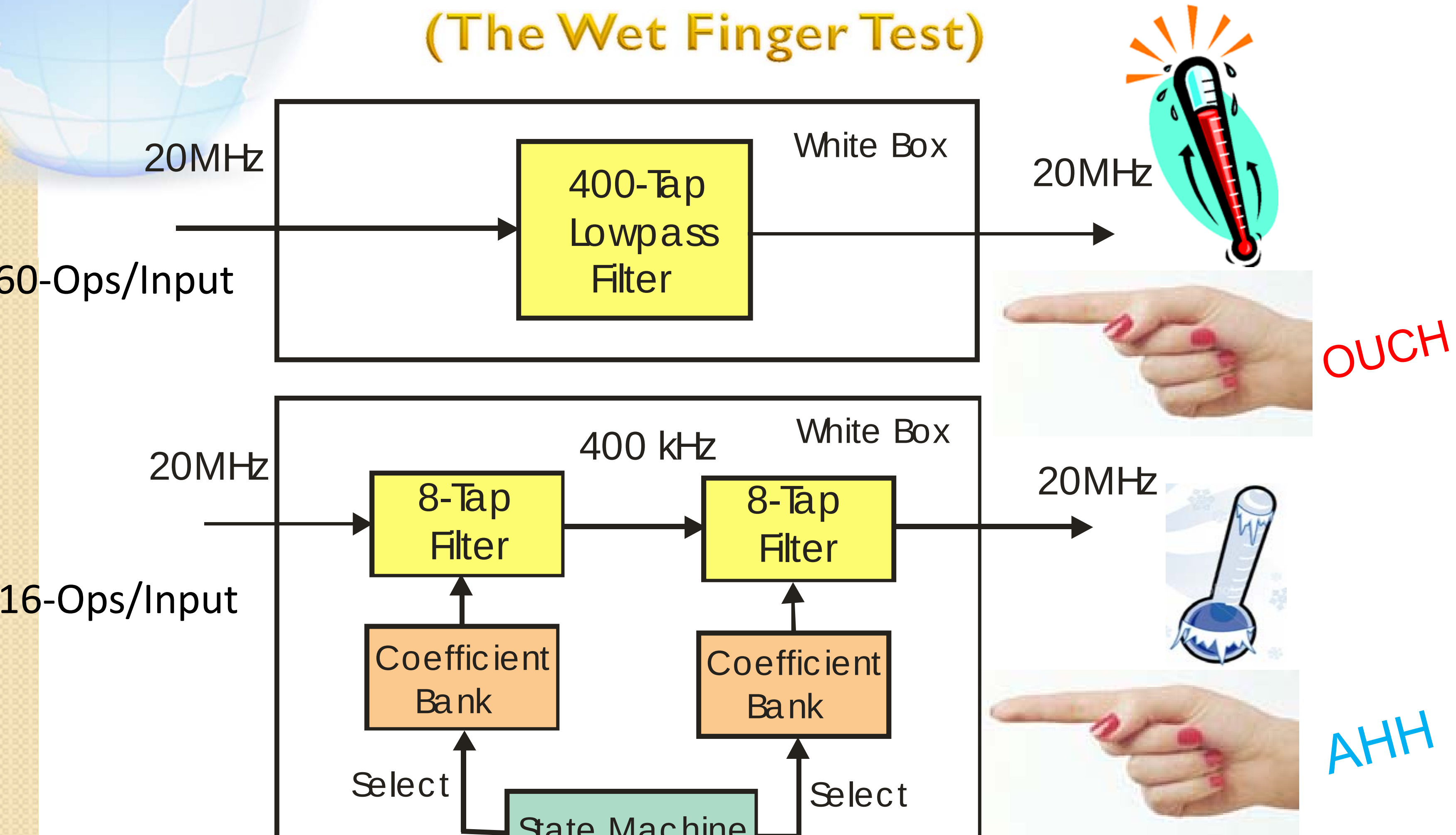
20 MHz

Efficient Polyphase Filter Implementation



Two Processing in Boxes:
ow can you tell which is which from outside box?

(The Wet Finger Test)



20MHz

60-Ops/Input

White Box

400-Tap Lowpass Filter

20MHz

OUCH

20MHz

16-Ops/Input

400 kHz

White Box

8-Tap Filter

8-Tap Filter

Coefficient Bank

Coefficient Bank

Select

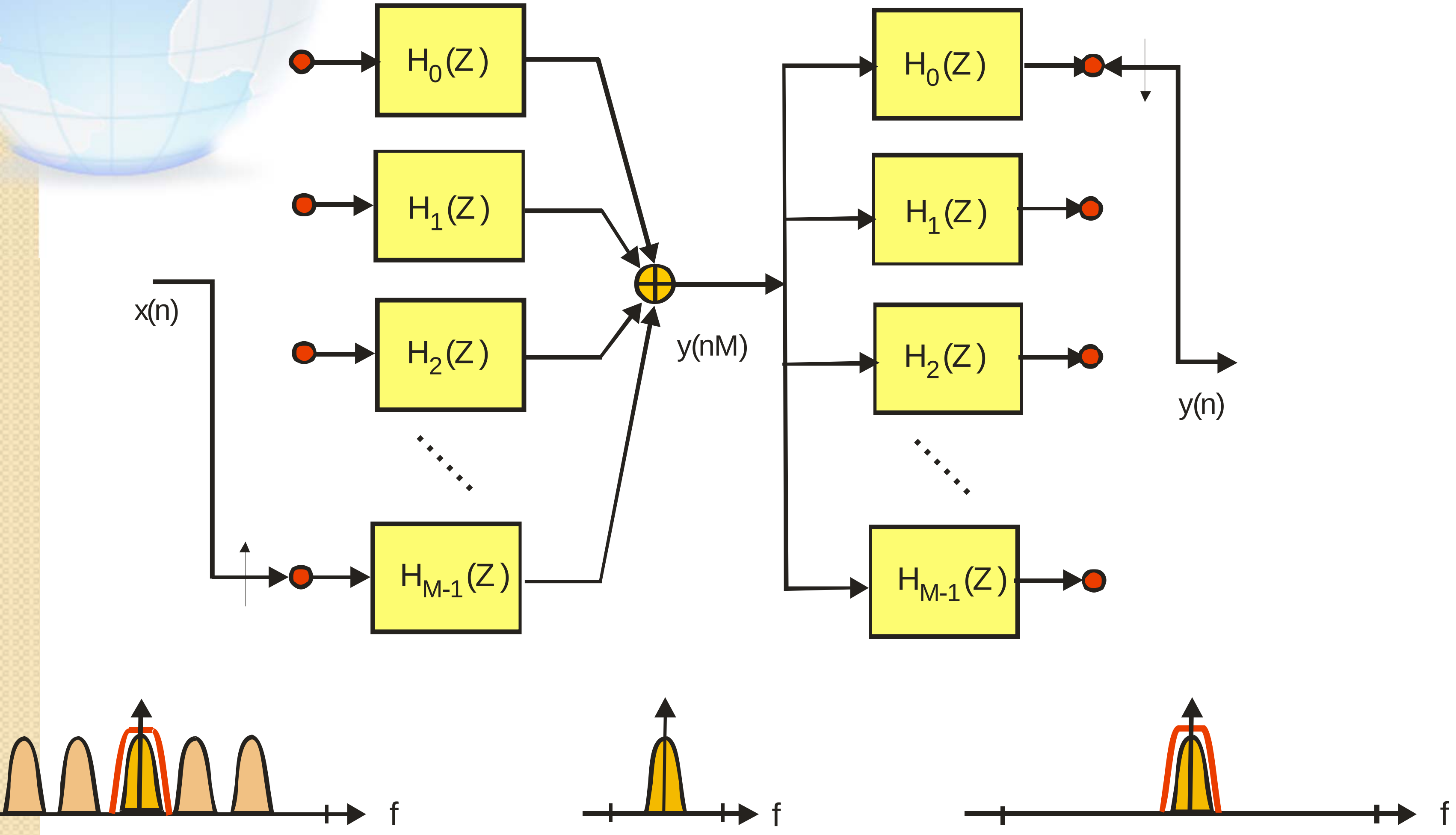
State Machine

Select

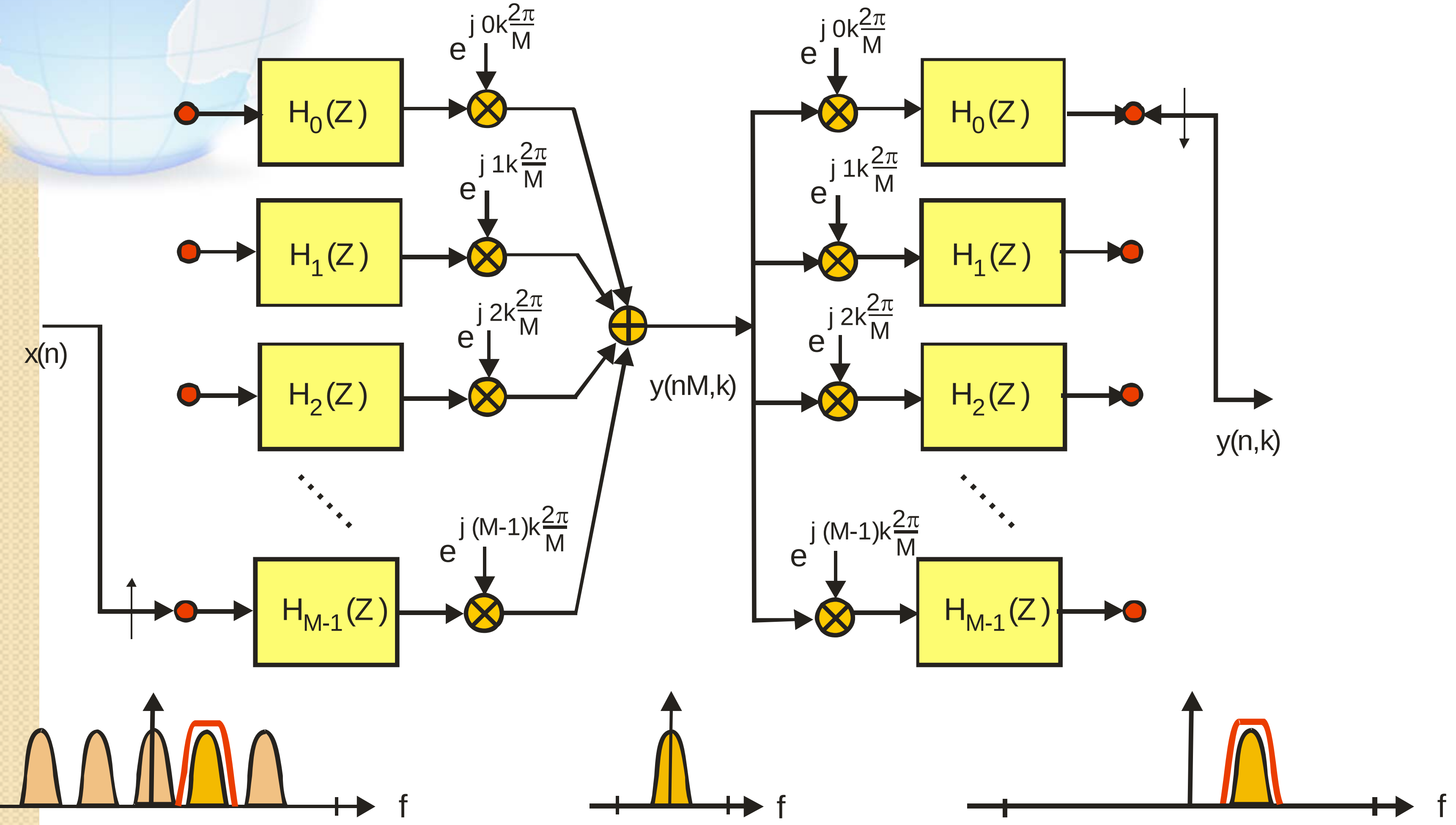
20MHz

AHH

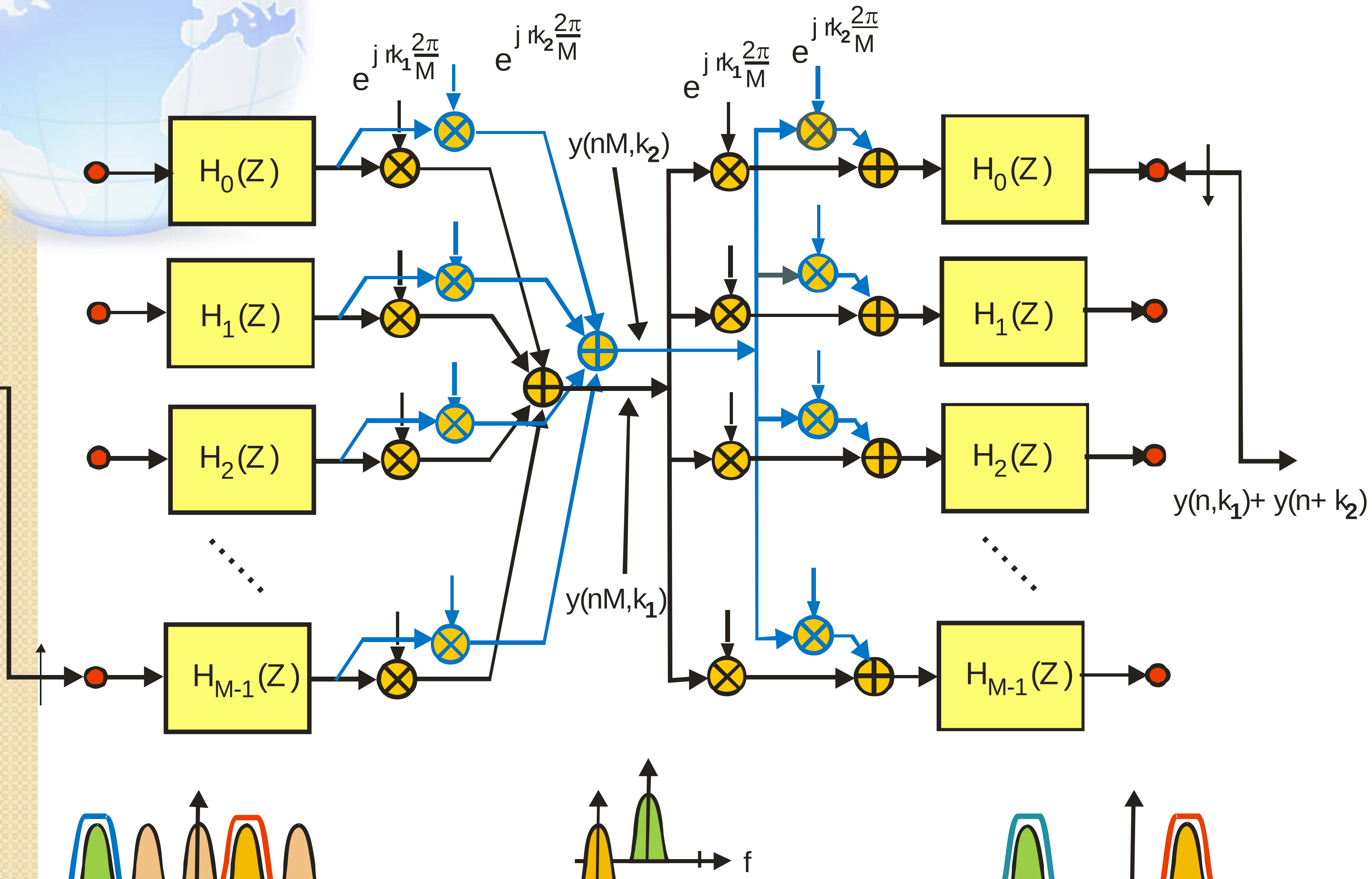
Polyphase Partition of Low-Pass Filter



Polyphase Partition of Band Pass Filter



lyphase Partition of Two Band Pass Filters

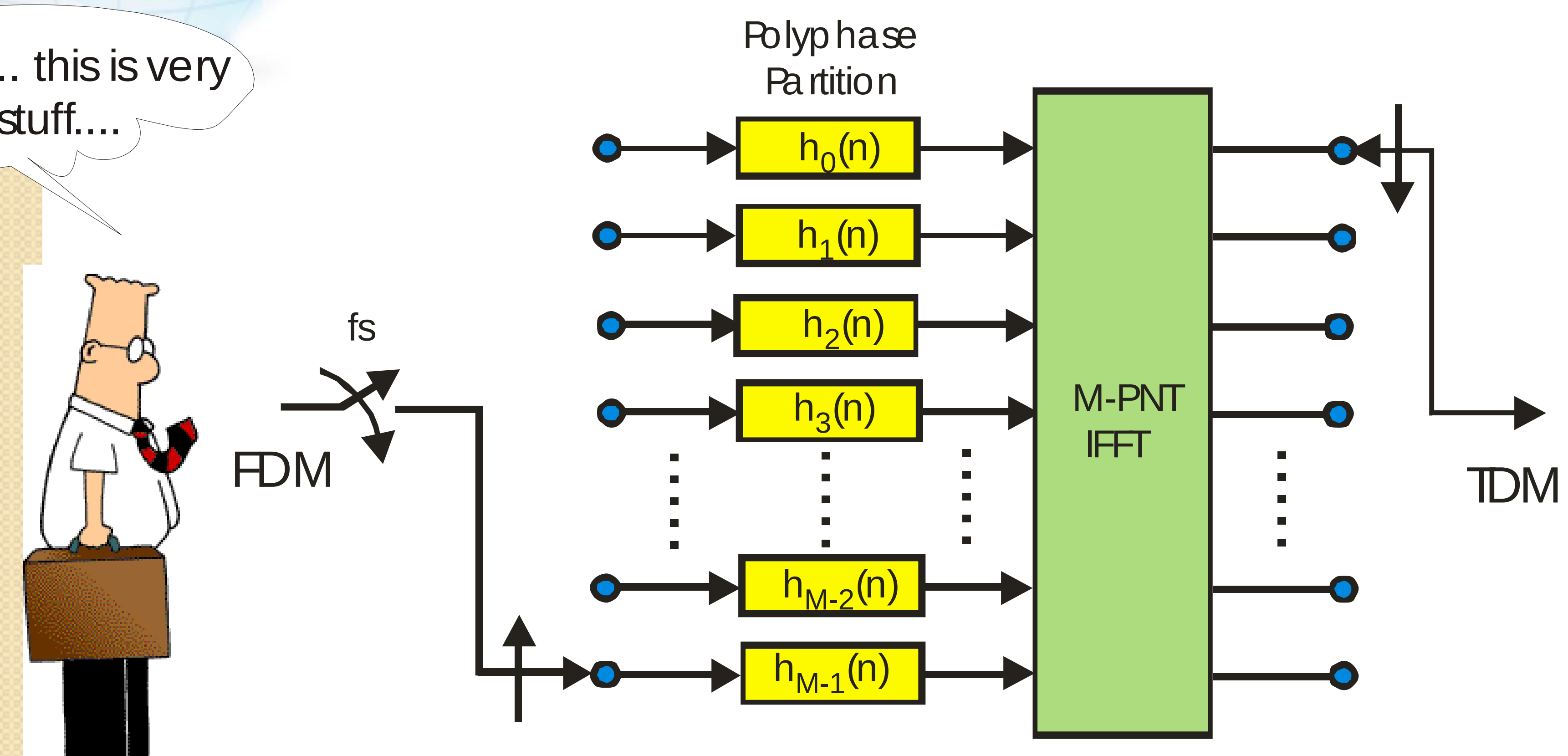


Vorkload for Multiple M-Path Filters

- 1-Channel M-to-1 Down Sample
  - 1-Filter and M Complex Phase Rotators
- 2-Channels M-to-1 Down Sample
  - 1-Filter and 2M Complex Phase Rotators
- K-Channels M-to-1 Down Sample
  - 1-Filter and kM Complex Phase Rotators
- M-channels M-to-1 Down Sample (use FFT)
  - 1-Filter and [$\log_2(M)/2$]M Complex Phase Rotators

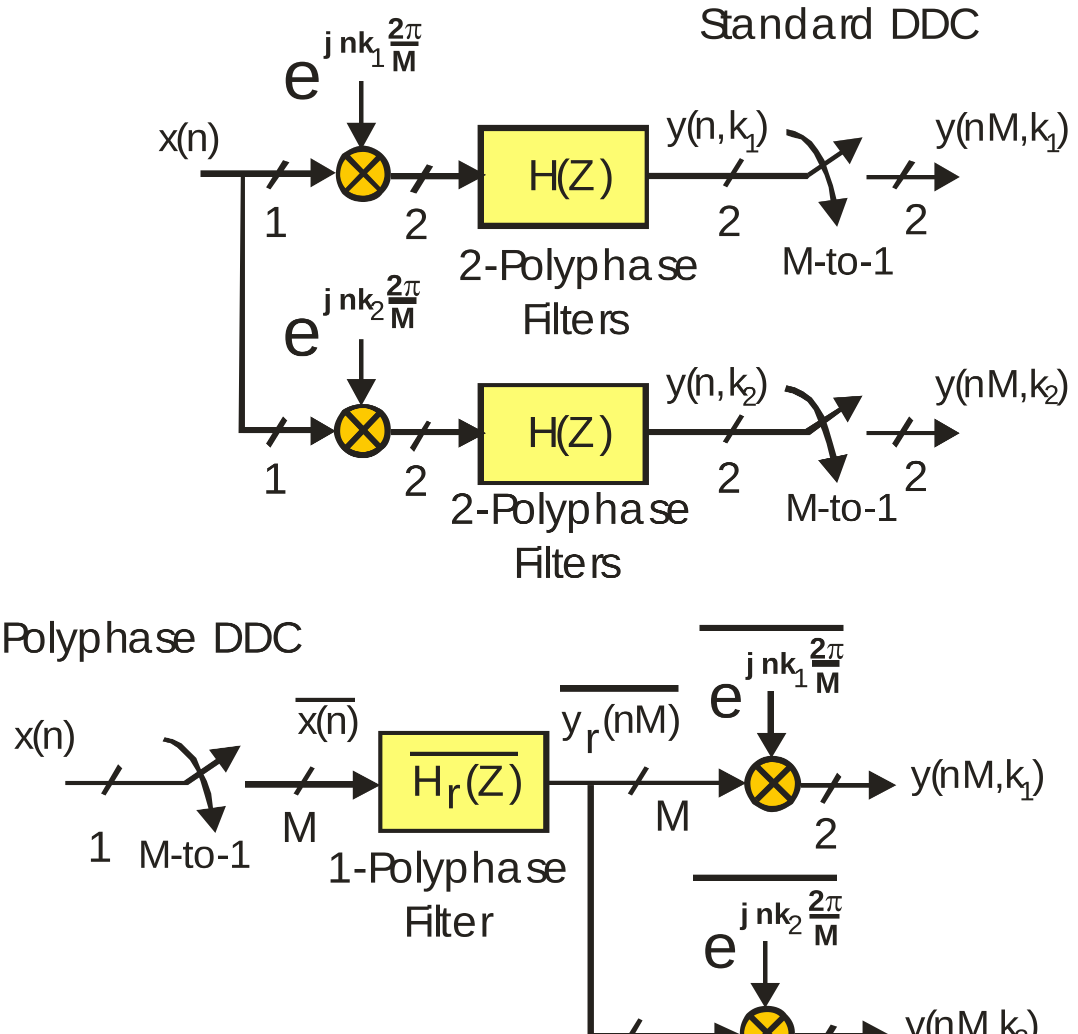
en k > $\mathrm{Log}_2(M)/2$ Build all channels and discard the channels you don't need!
M=16, $\mathrm{Log}_2(16)/2 = 2$: thus if you want 2 or more, Build them all!
M=128, $\mathrm{Log}_2(128)/2 = 3.5$: thus if you want 4 or more, Build them all!
M=1024, $\mathrm{Log}_2(1024)/2 = 5$: thus if you want 5 or more, Build them all!

M-Channel Channelizer: Resampled M-Path Narrowband Filter
Channels Alias to Baseband: Phase Aligned Sums Separate Aliases:
rk Performed at Low Output Rate Rather Than at High Input Rate.
One Input Filter Services M-Output Channels

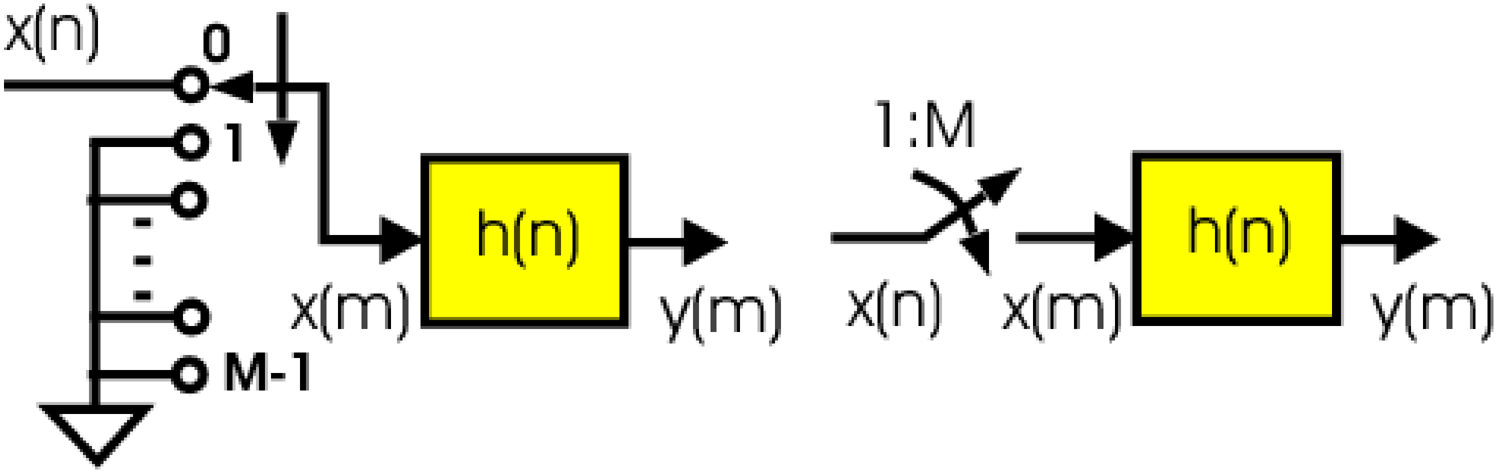


$h_r(n)= h(r+ nM)$

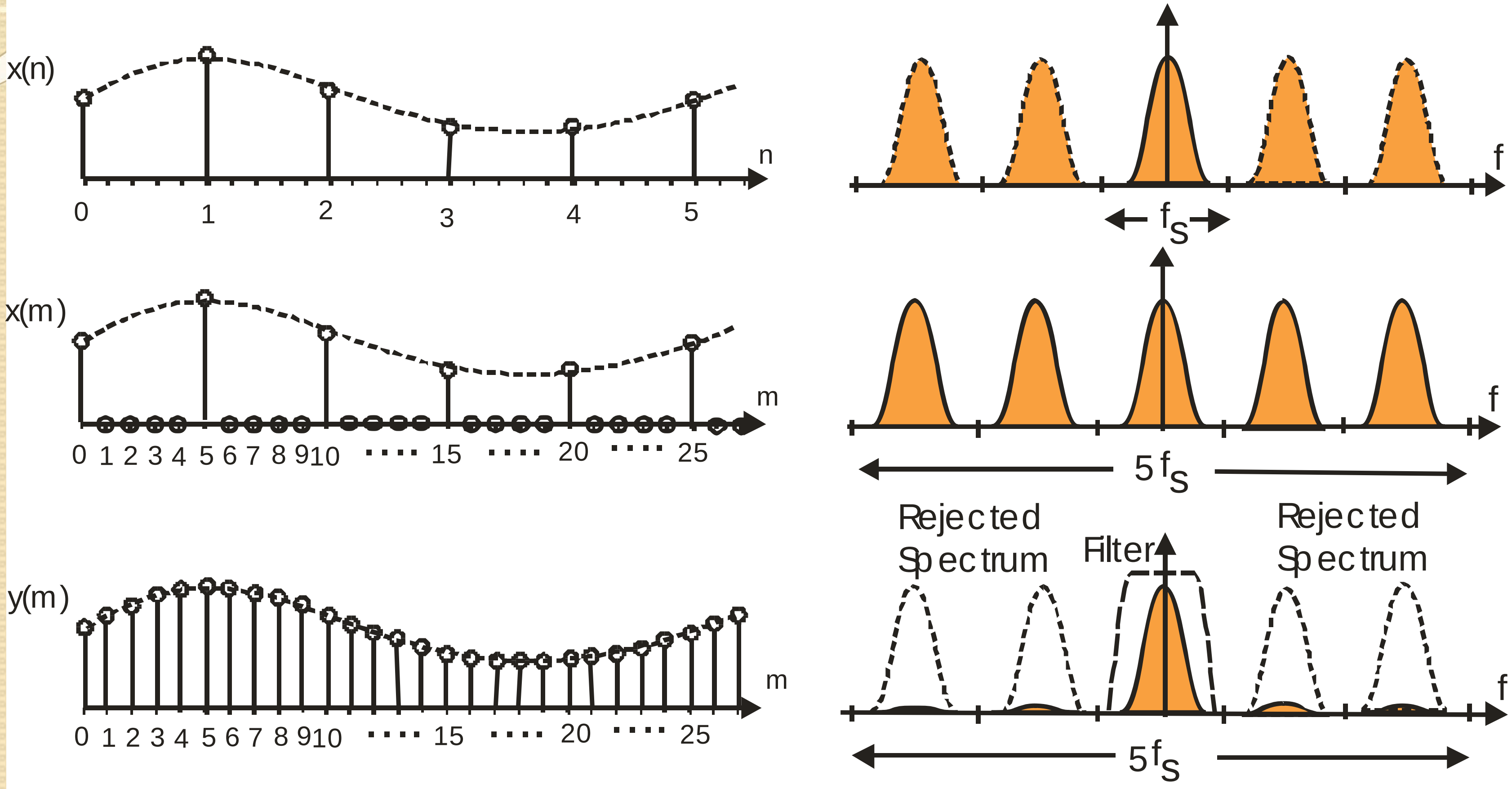
Dual Channel Armstrong and Multirate Aliased Polyphase Receiver



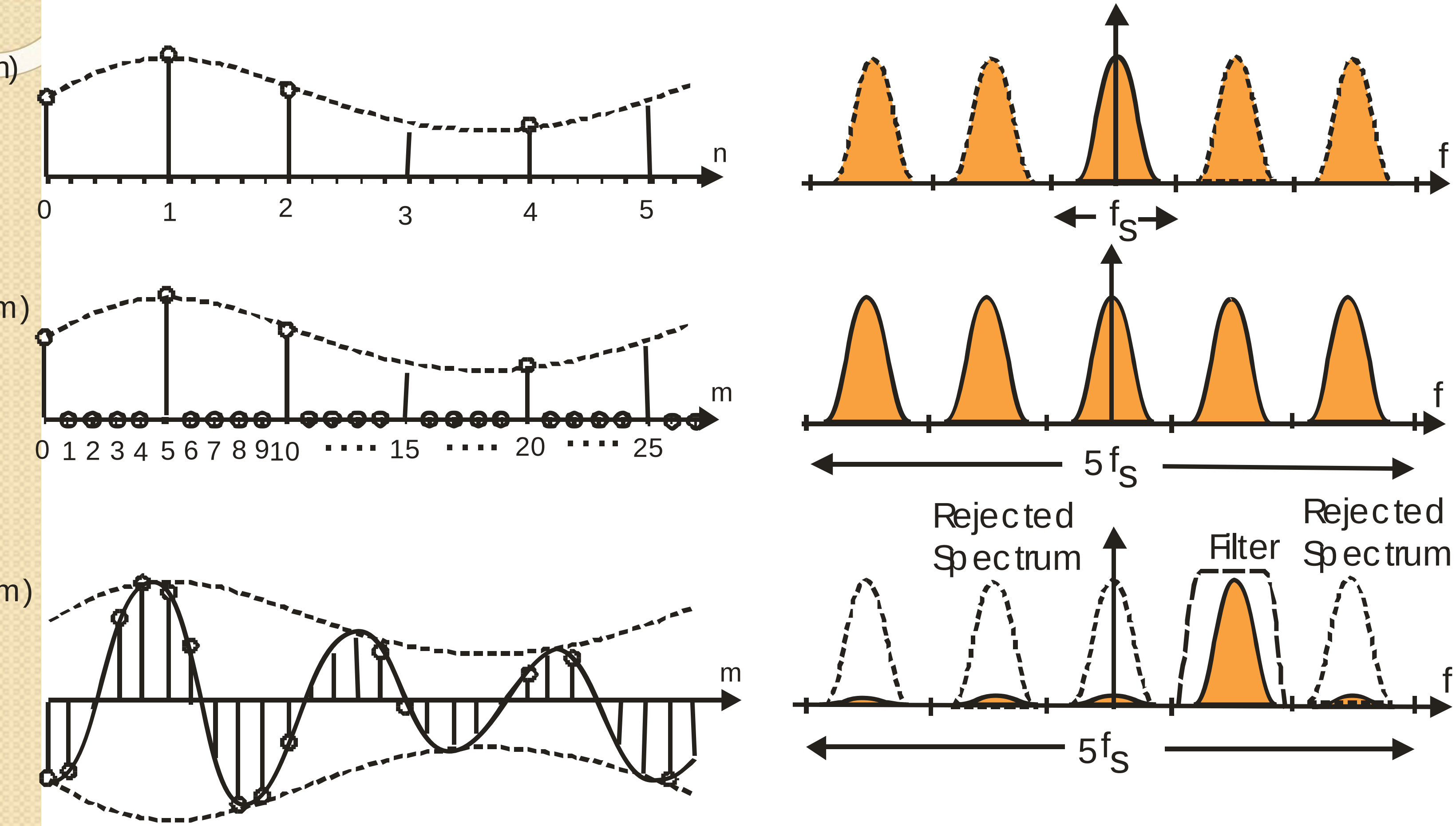
Up-sampling by Zero Packing and Filtering



Spectra Of Input, of Zero-Packed, and of Low-Pass Filtered Zero-Packed Signal

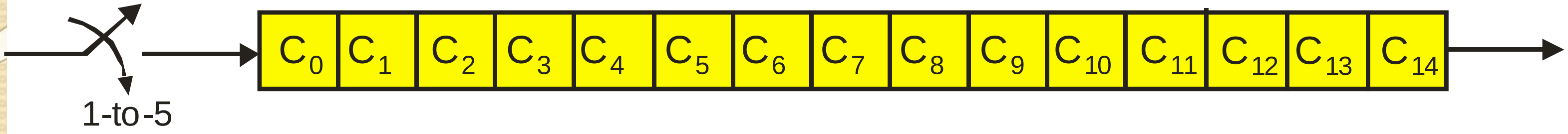


Spectra Of Input, of Zero-Packed, and of Band Pass Filtered Zero-Pack Signal

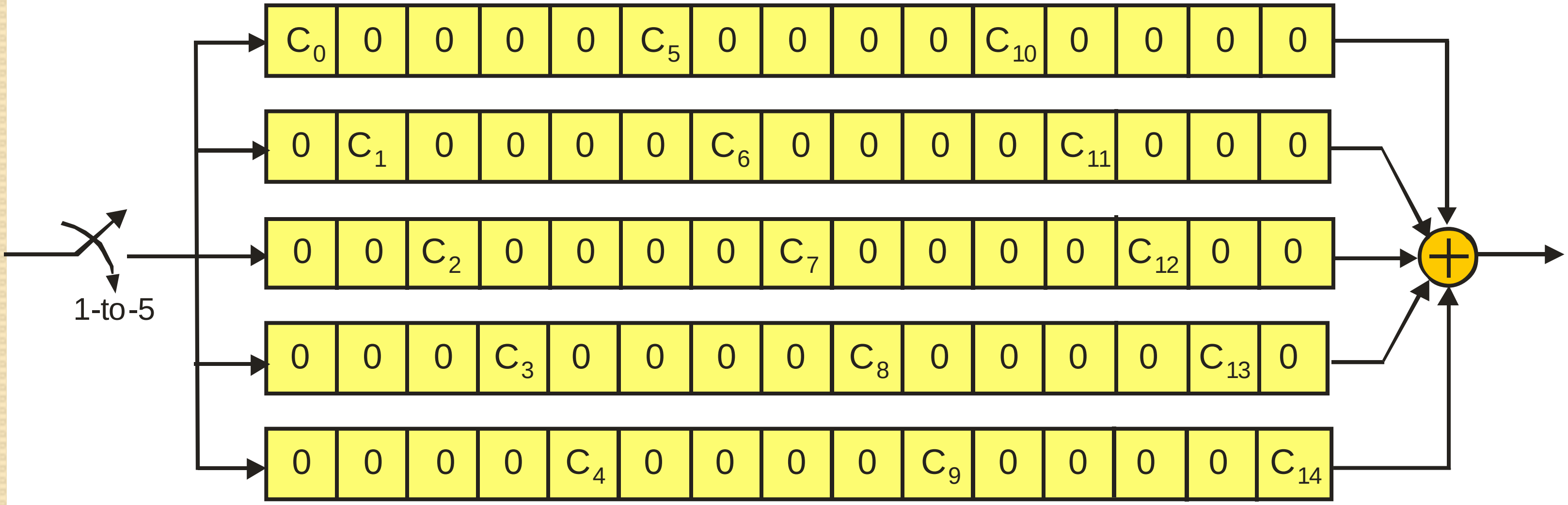


Polyphase Partition of Resampling Filter

$$H(Z)=\sum_{n=0}^{N-1} h(n)Z^{-n}$$

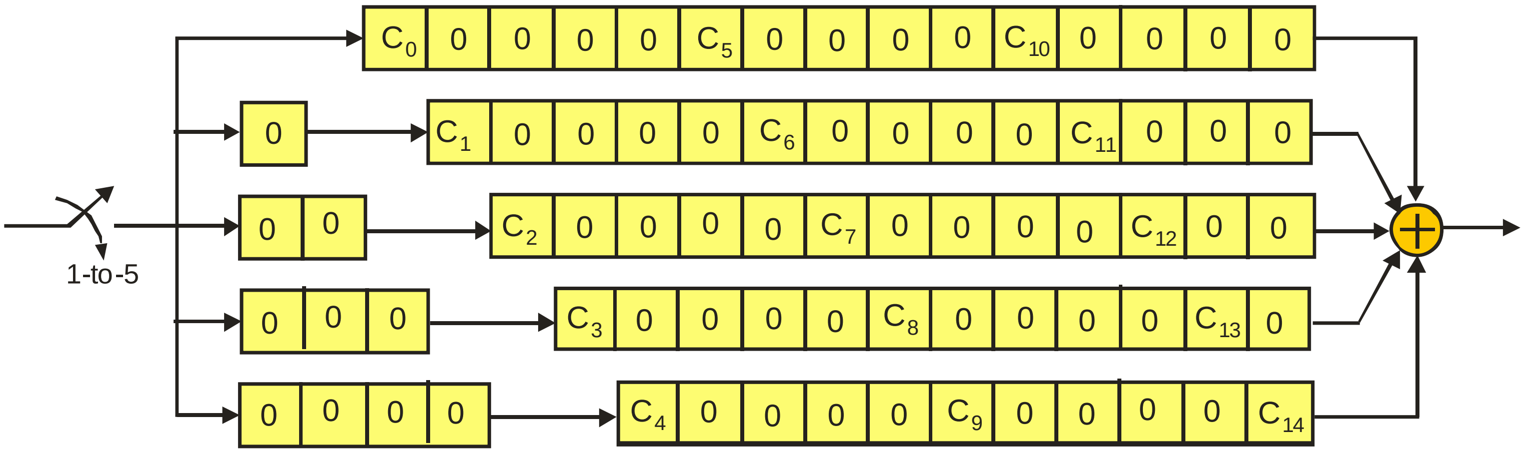


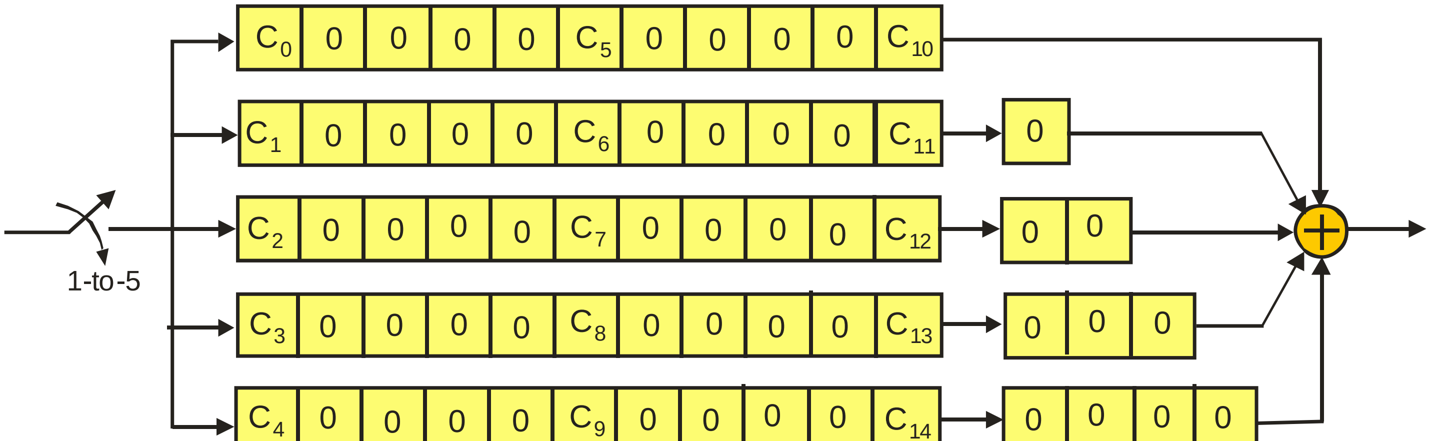
$$H(Z)=\sum_{r=0}^{M-1}\sum_{n=0}^{\frac{N}{M}-1} h(r+nM)Z^{-(r+nM)}$$



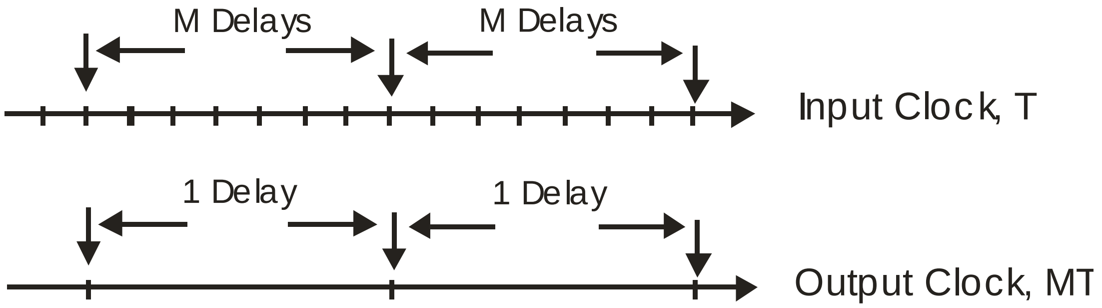
Factor Delays and Rearrange

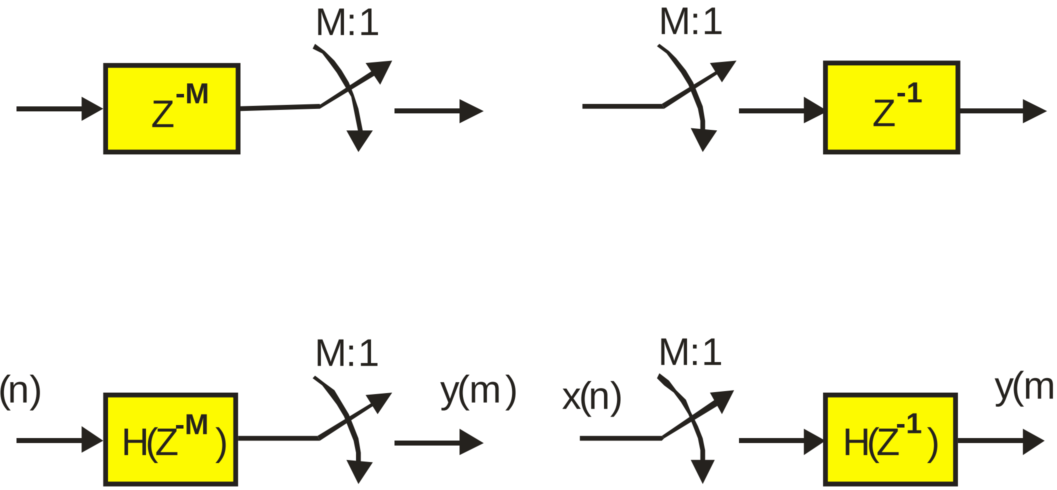
$$H(Z)=\sum_{r=0}^{M-1} Z^{-r}\sum_{n=0}^{\frac{N}{M}-1} h(r+nM)Z^{-nM}$$



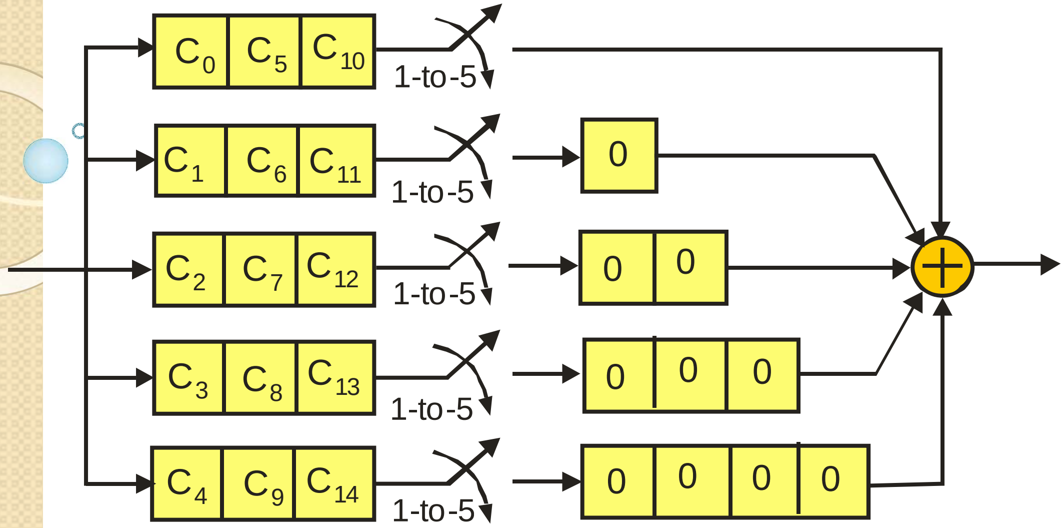


Noble Identity:
Interchange M-Delays with M-to-1 Resample

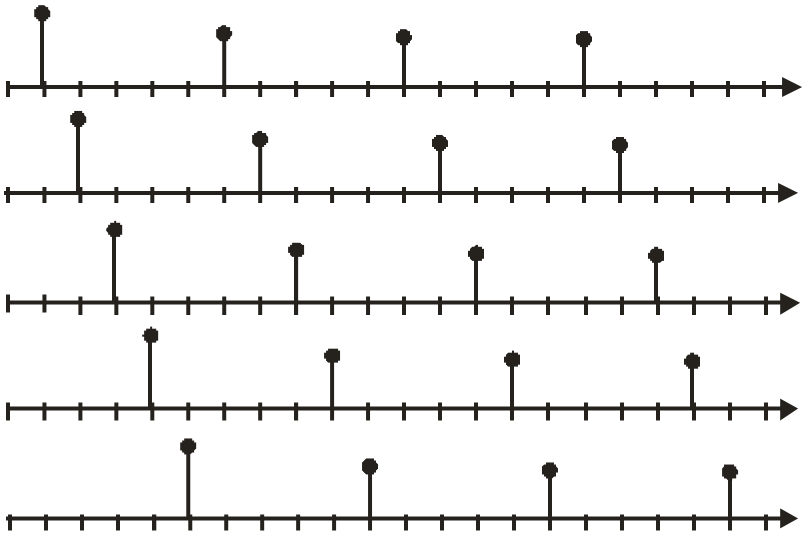


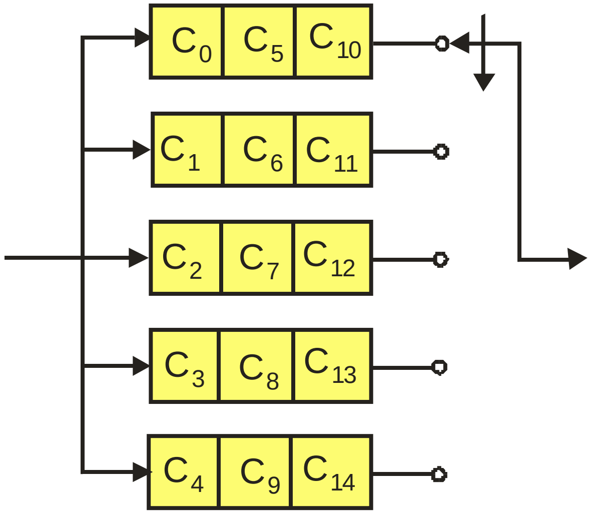


Interchange Filter and Resampler



Replace Up Samplers, Delays, and Summer with M-Port Output Commutator





Low-Pass Replaced by Band-Pass

$$G(Z)=\sum_{n=0}^{N-1}h(n)e^{j\frac{2\pi}{M}kn}Z^{-n}$$

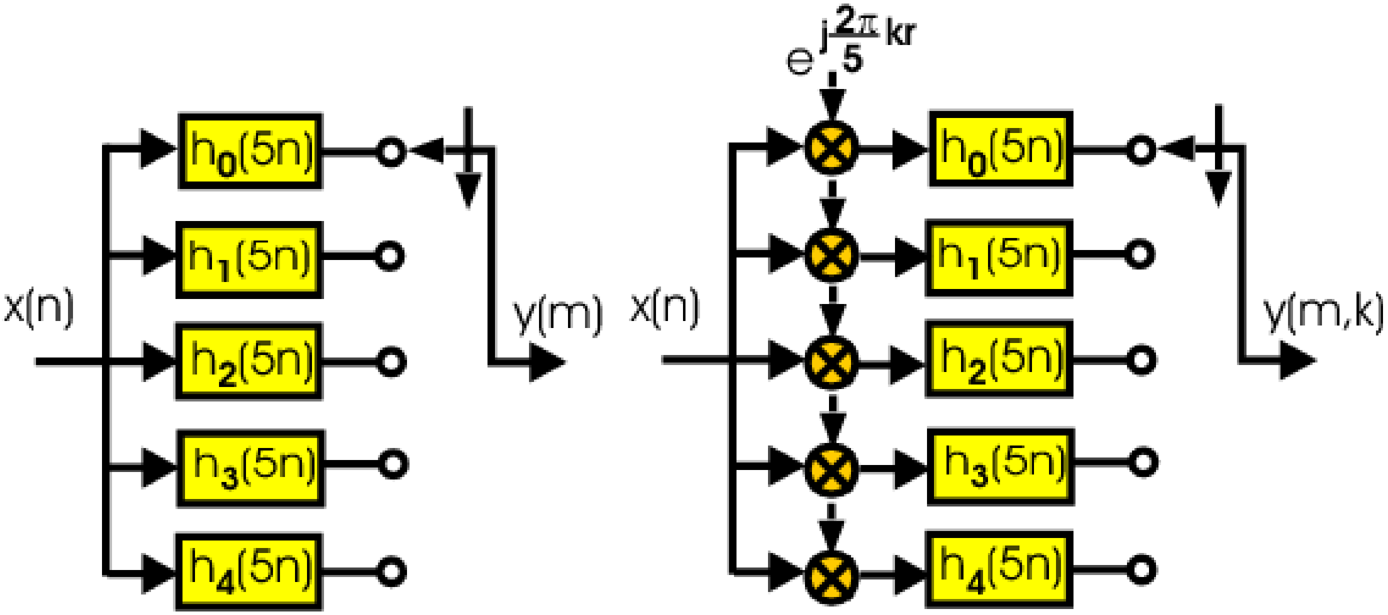
$$G(Z)=\sum_{r=0}^{M-1}\sum_{n=0}^{\frac{N}{M}-1}h(r+nM)e^{j\frac{2\pi}{M}(r+nM)k}Z^{-(r+nM)}$$

$$G(Z)=\sum_{r=0}^{M-1}Z^{-r}e^{j\frac{2\pi}{M}rk}\sum_{n=0}^{\frac{N}{M}-1}h(r+nM)e^{j\frac{2\pi}{M}nM}Z^{-nM}$$

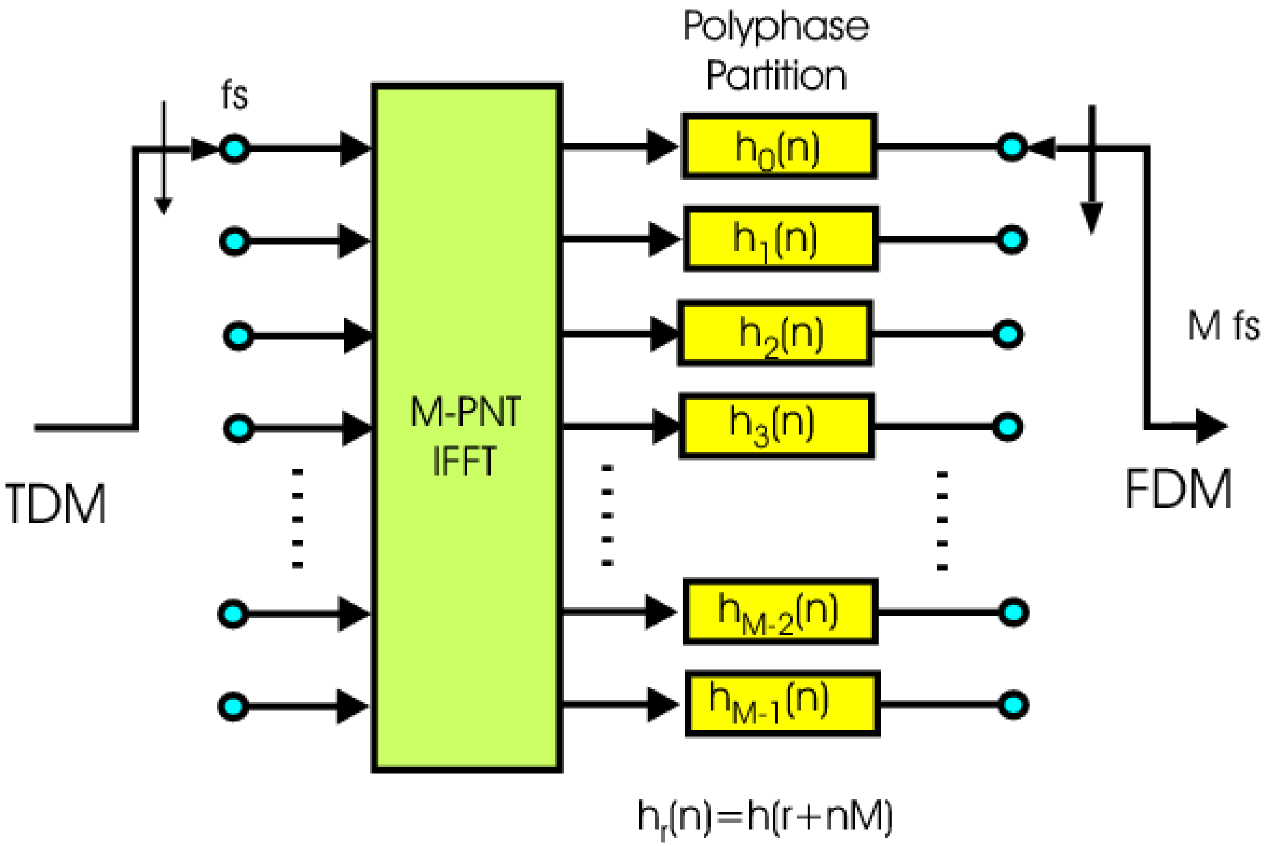
$$G(Z)=\sum_{r=0}^{M-1}Z^{-r}e^{j\frac{2\pi}{M}rk}\sum_{n=0}^{\frac{N}{M}-1}h(r+nM)Z^{-nM}$$

Spin The Delays,
Don’t Touch the M-Path Partitioned Weights

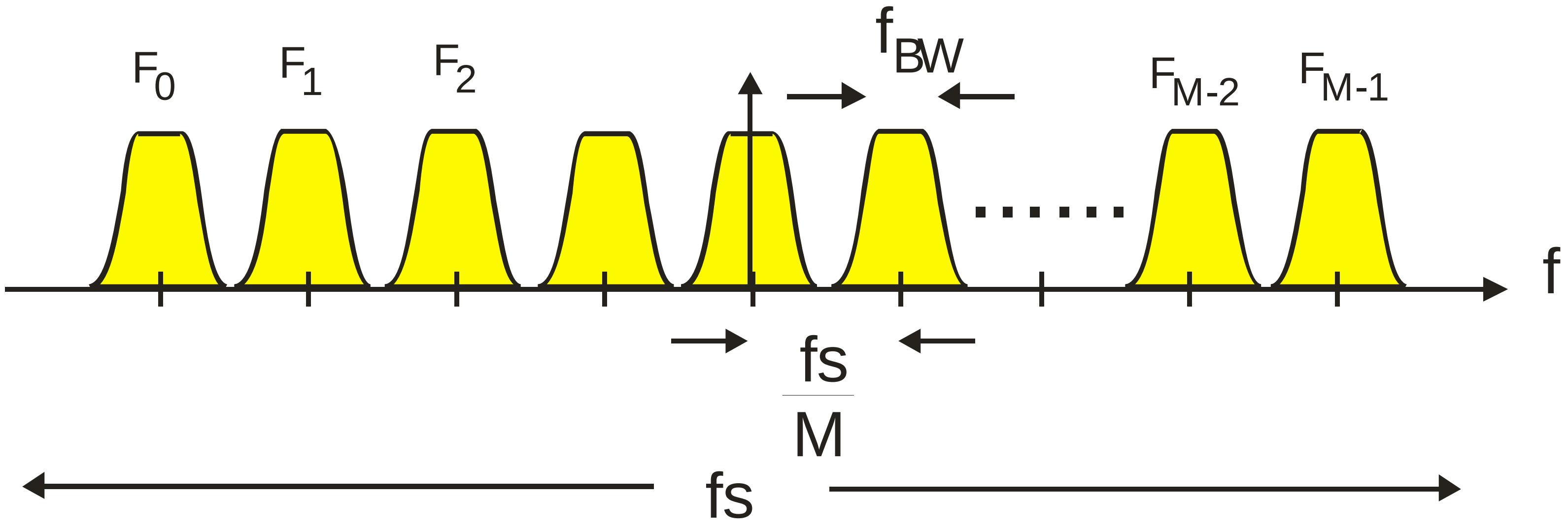
Low-Pass to Band-Pass
1-to-M Up-Sampling Filter



M-Path, M-Channel Channelizer: Spinners are in IFFT



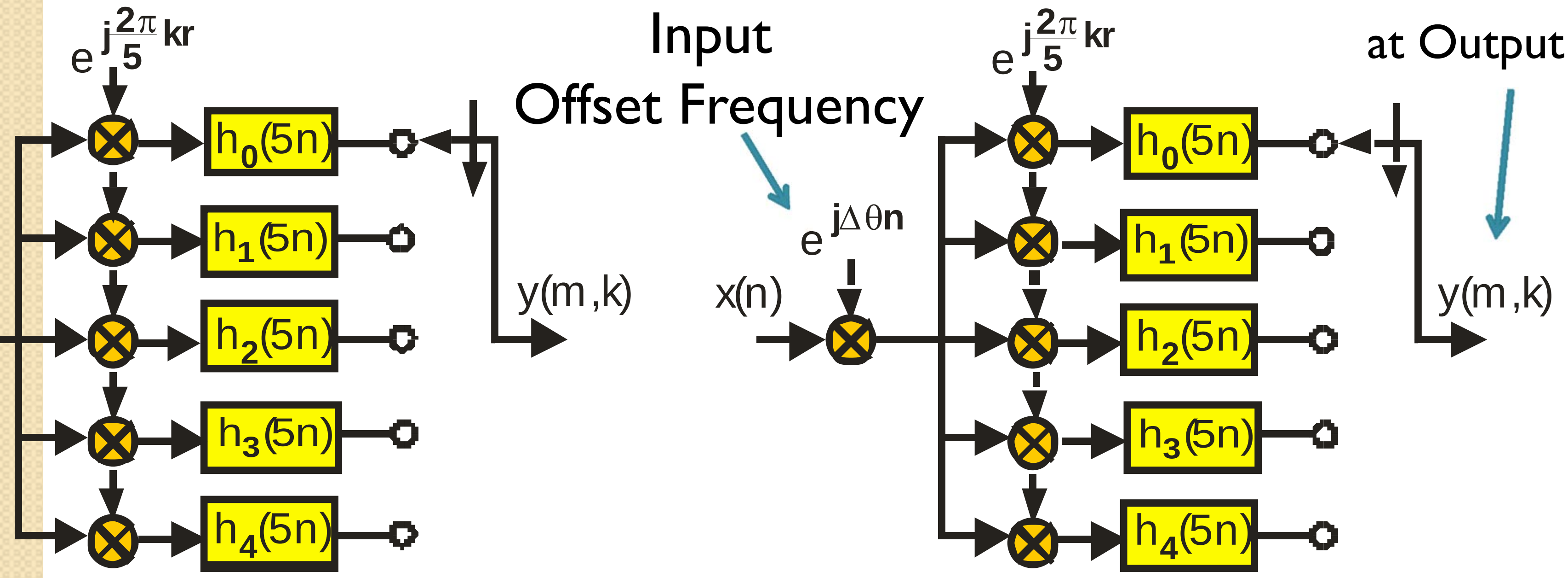
M-Point IFFT Supplies Phase Spinners to Form Up Converters to all Multiples of Input Sample Rate



All Output Channels Centered on Multiples of Input Sample Rate

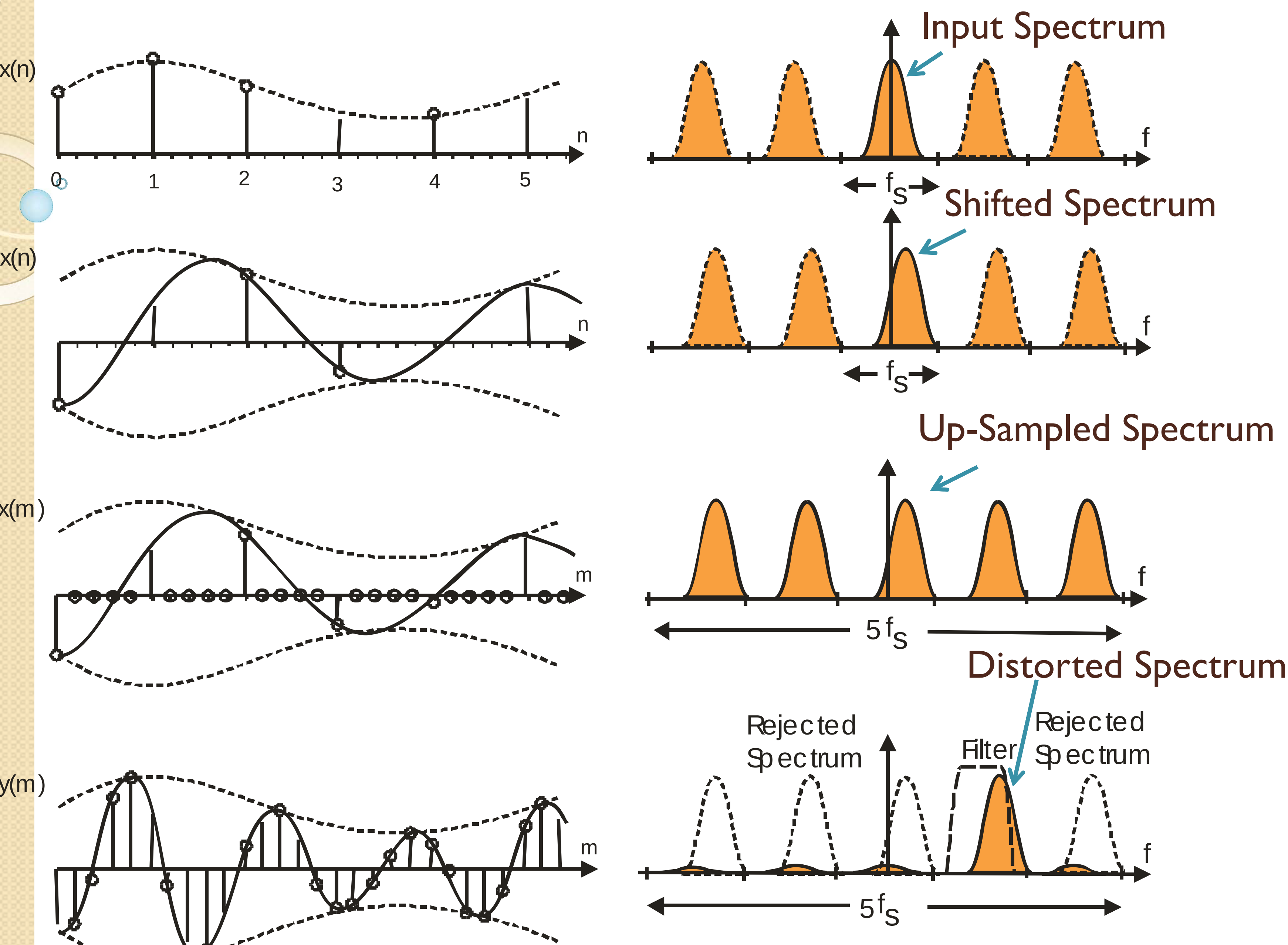
Example: Multiples of 6-MHz

Heterodyne Input Signal a Small Frequency Offset from DC: Channelizer Aliases DC to Channel Center and offset signal from DC is Offset from Channel Center



Input
Offset Frequency

Input Offset
Observed
at Output


Input Spectrum

Shifted Spectrum

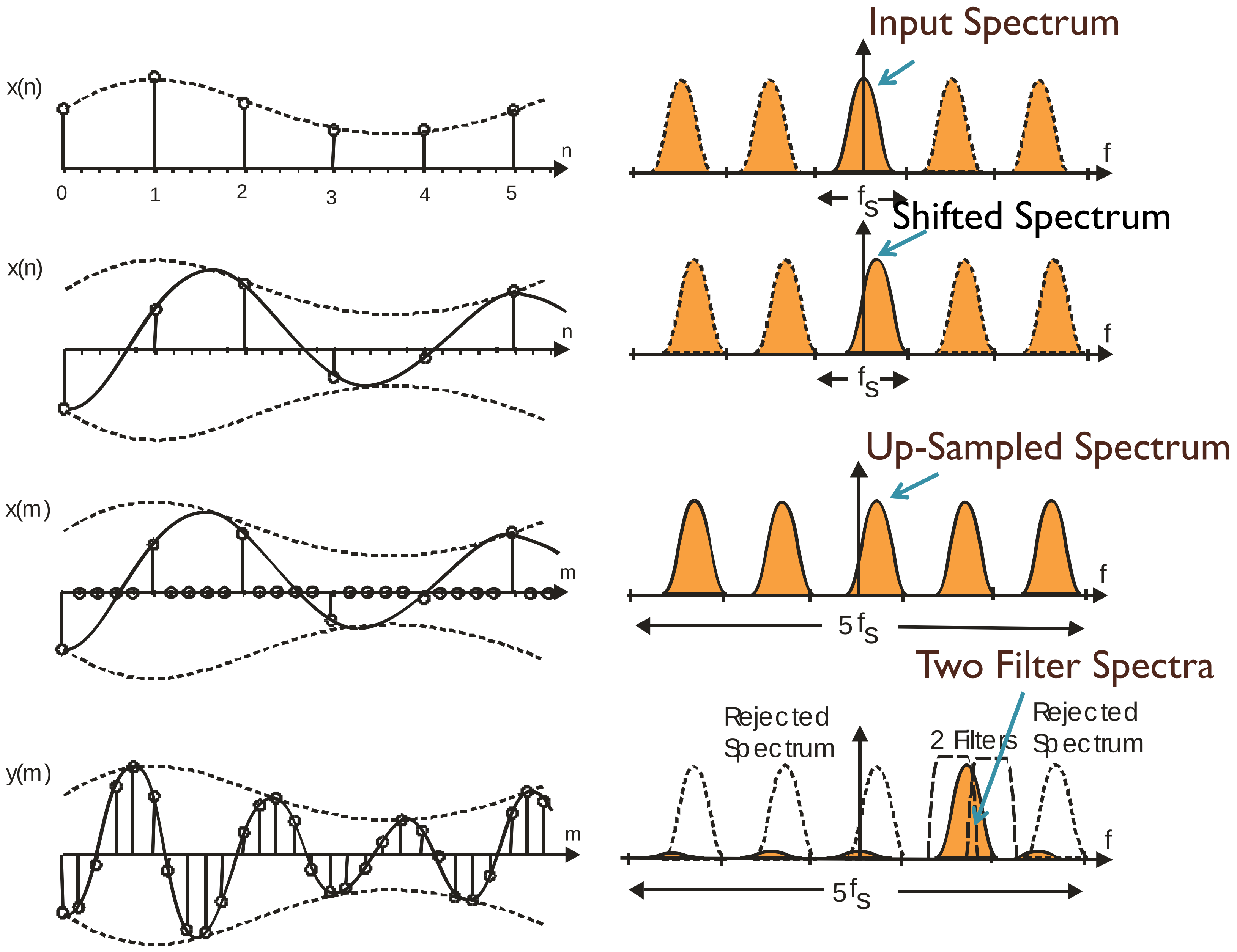
Up-Sampled Spectrum

Distorted Spectrum

Rejected Spectrum

Filter

Rejected Spectrum



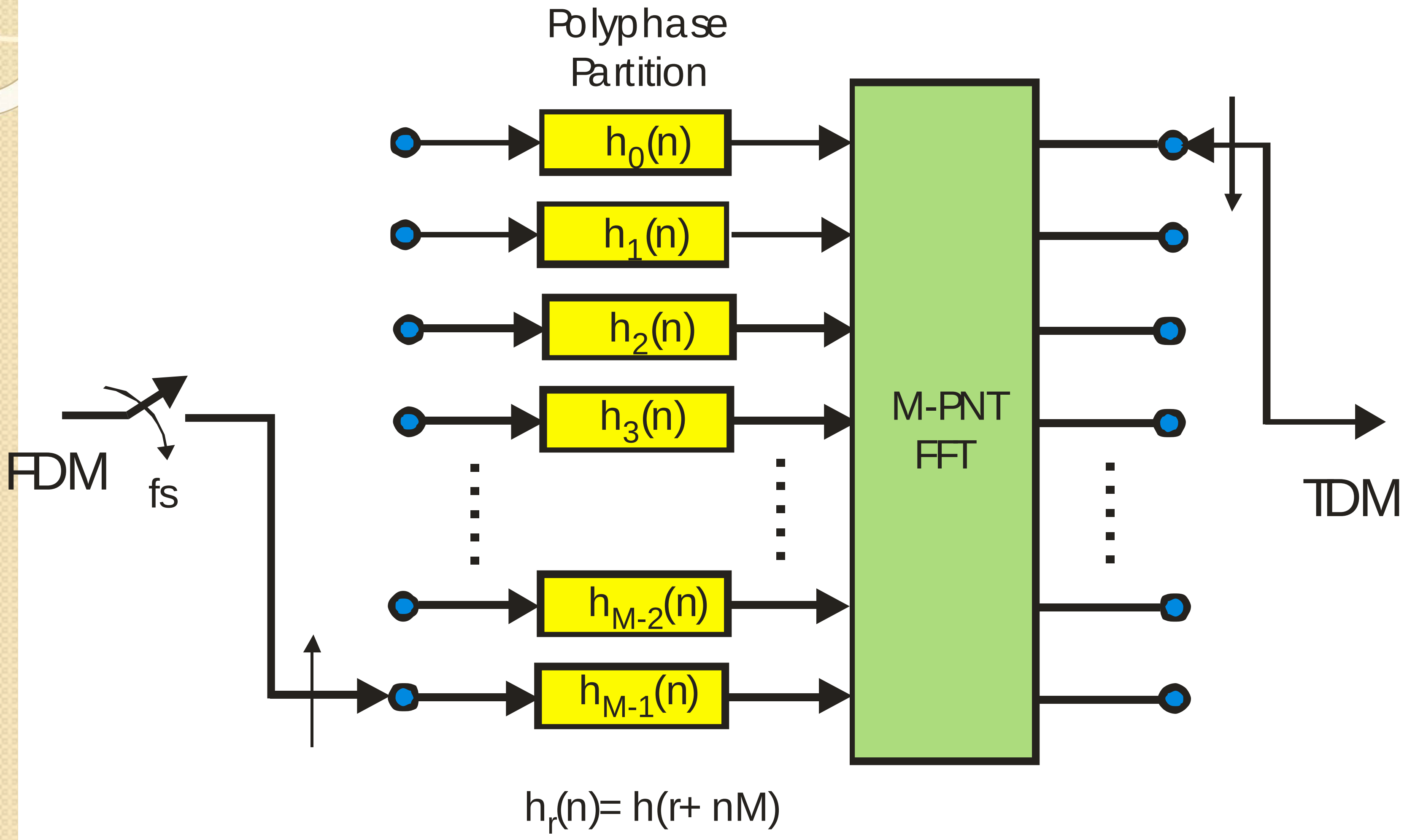
Input Spectrum

Shifted Spectrum

Up-Sampled Spectrum

Two Filter Spectra

M-Channel Polyphase Channelizer:
M-path Filter and M-point FFT



Various Filter-Channelizer Configurations

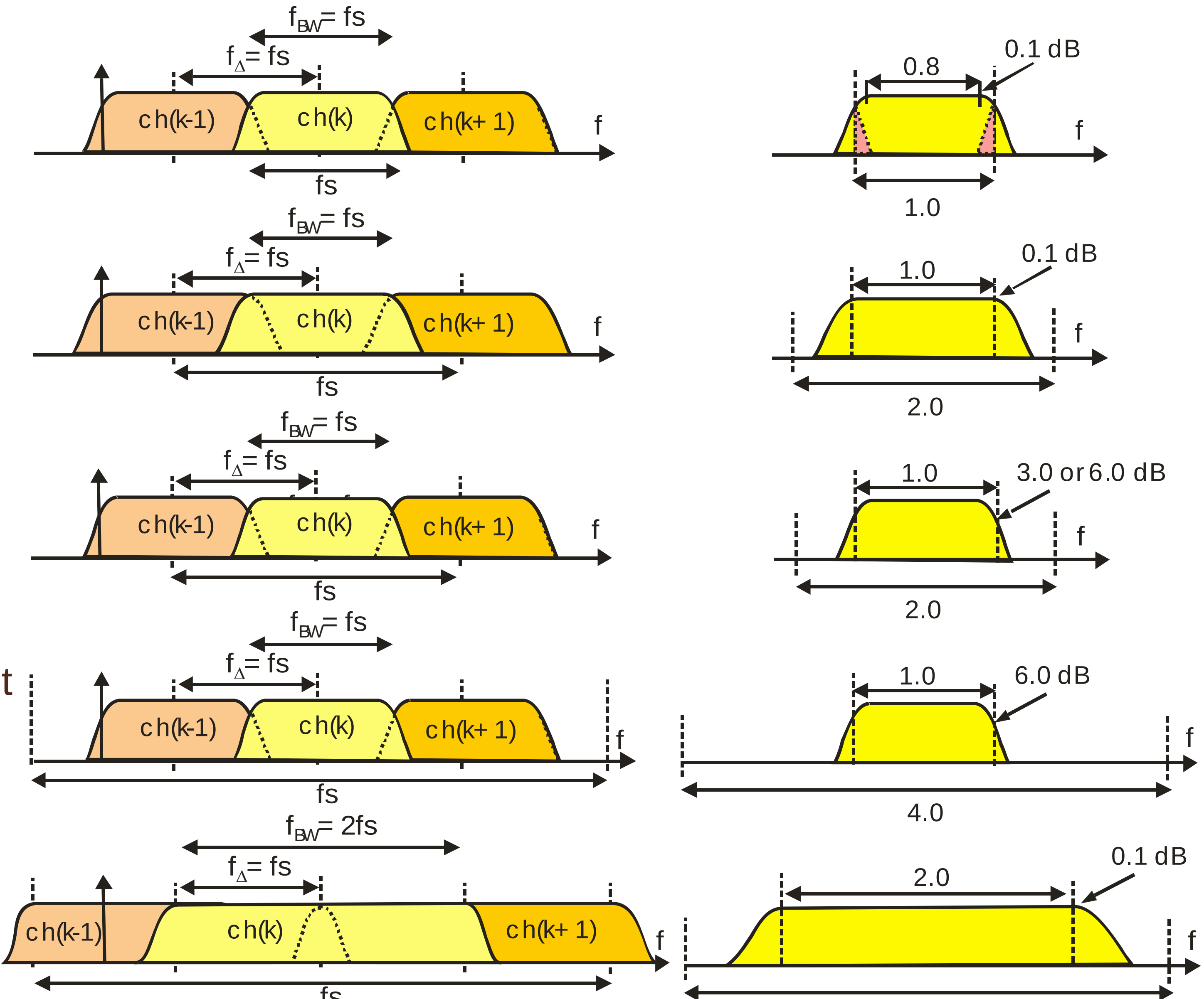
Critically
Sampled
$f_S=f_C$

Nyquist
Sampling
$f_S=2f_C$

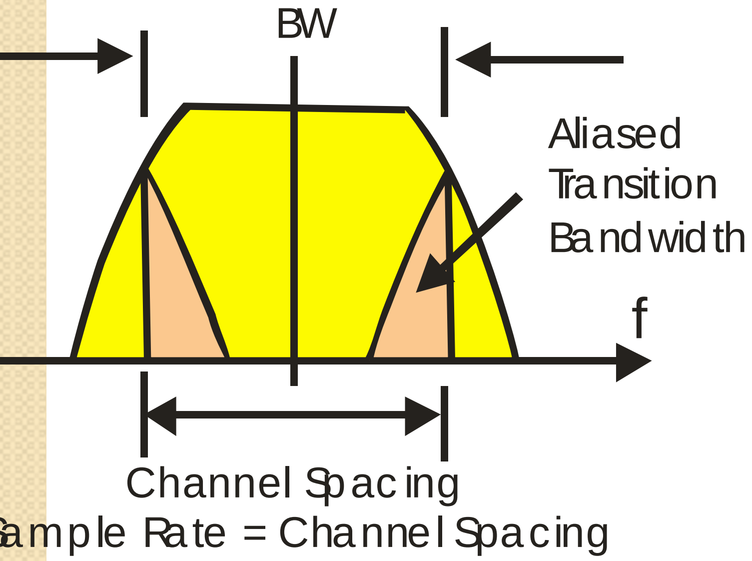
RT Nyquist
Filter
$f_S=2f_C$

QRT Nyquist
Filter
$f_S=4f_C$

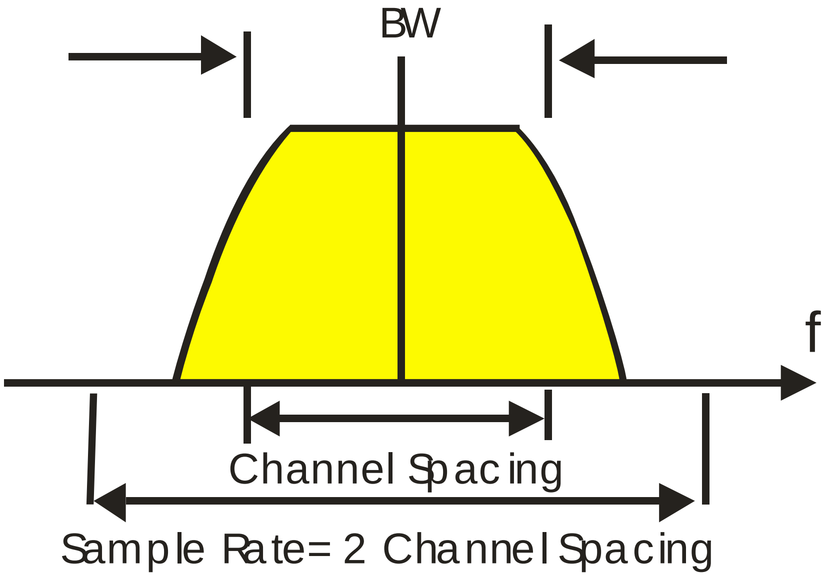
Nyquist
Sampling
$f_S=4f_C$



Filter Sampled at Rate to Avoid Band Edge Aliasing

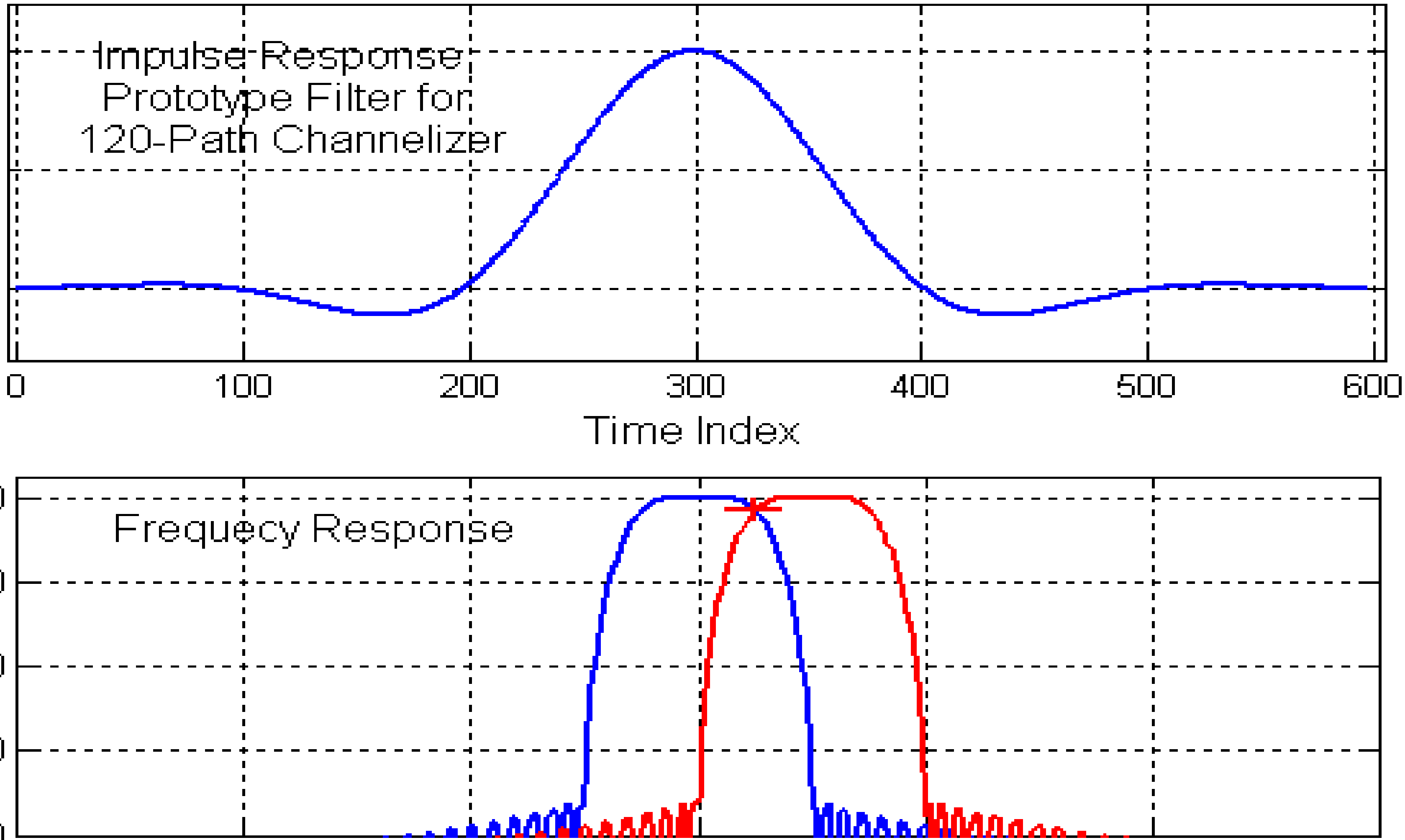


Sample Rate = Channel Spacing

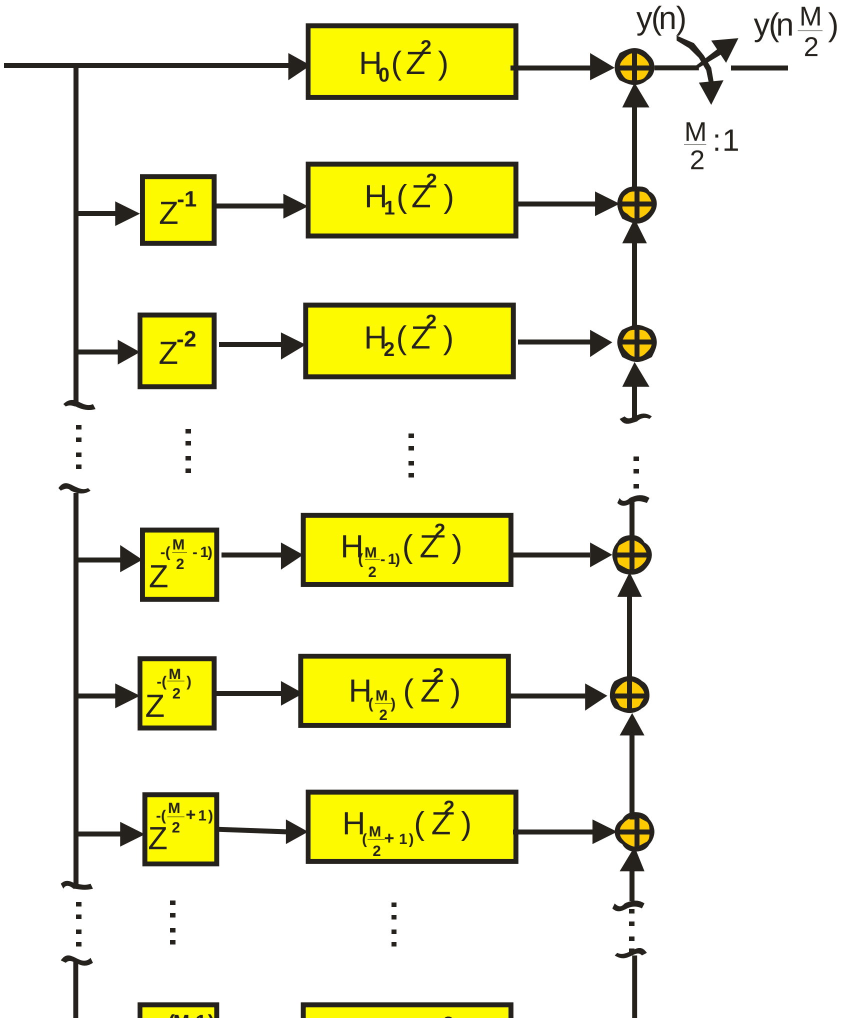


Sample Rate= 2 Channel Spacing

Prototype Low-Pass Filter for 120 Channel Channelizer

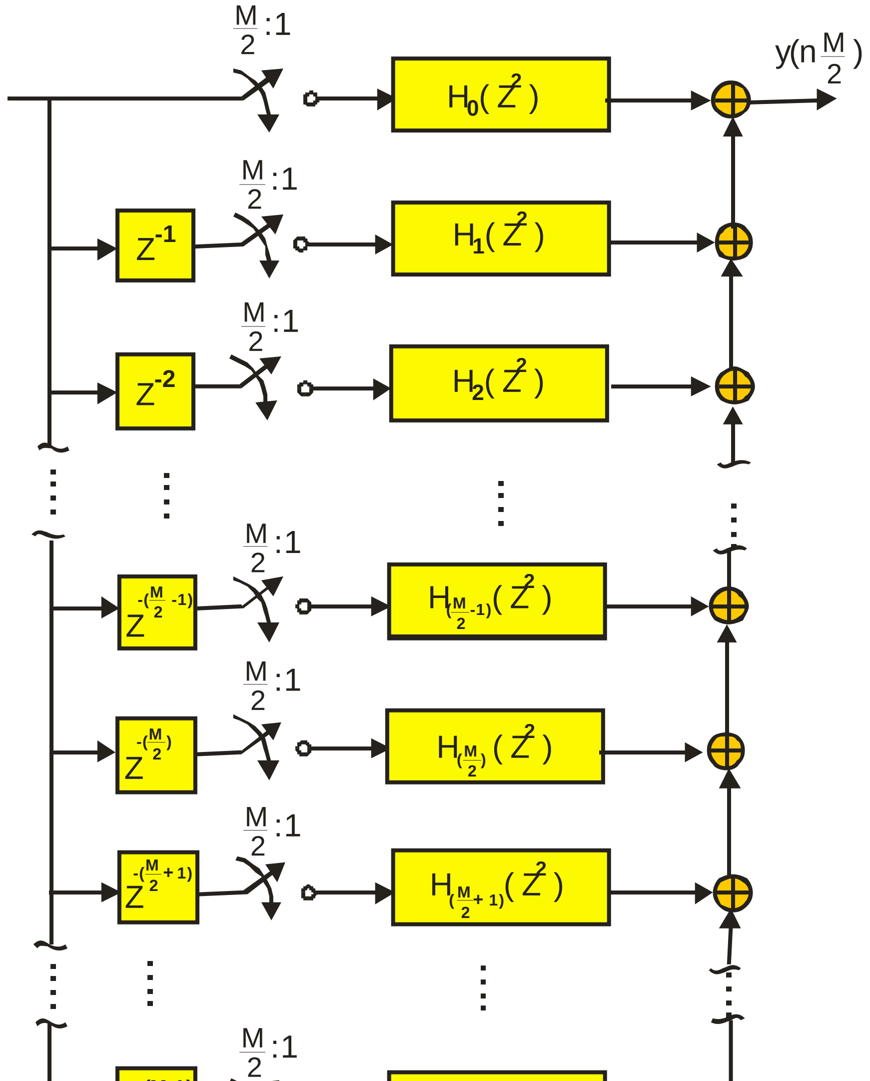


M-Path Polyphase Filter and M/2-to-1 Down Sampling

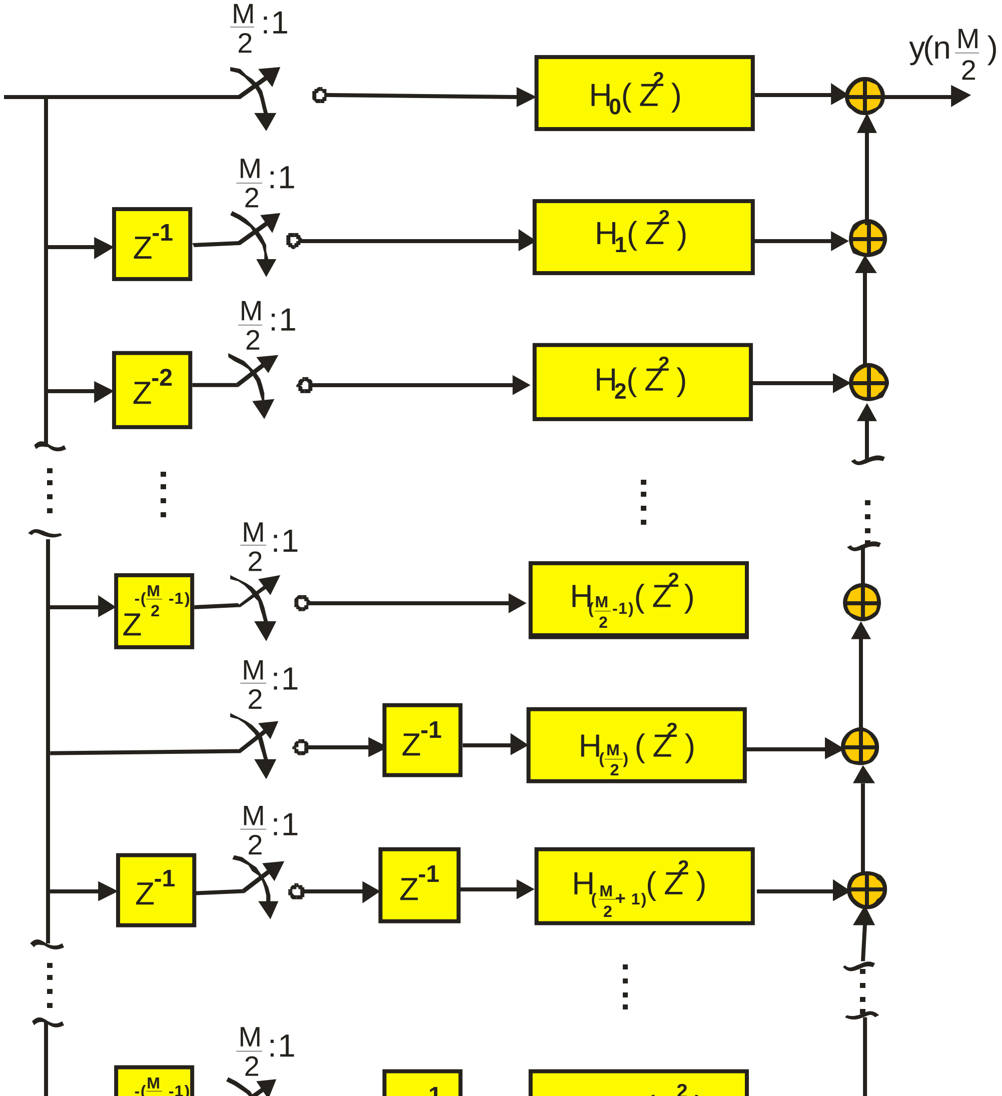


Use Noble Identity to Pull M/2-to-1 Resampler Through Path Filter

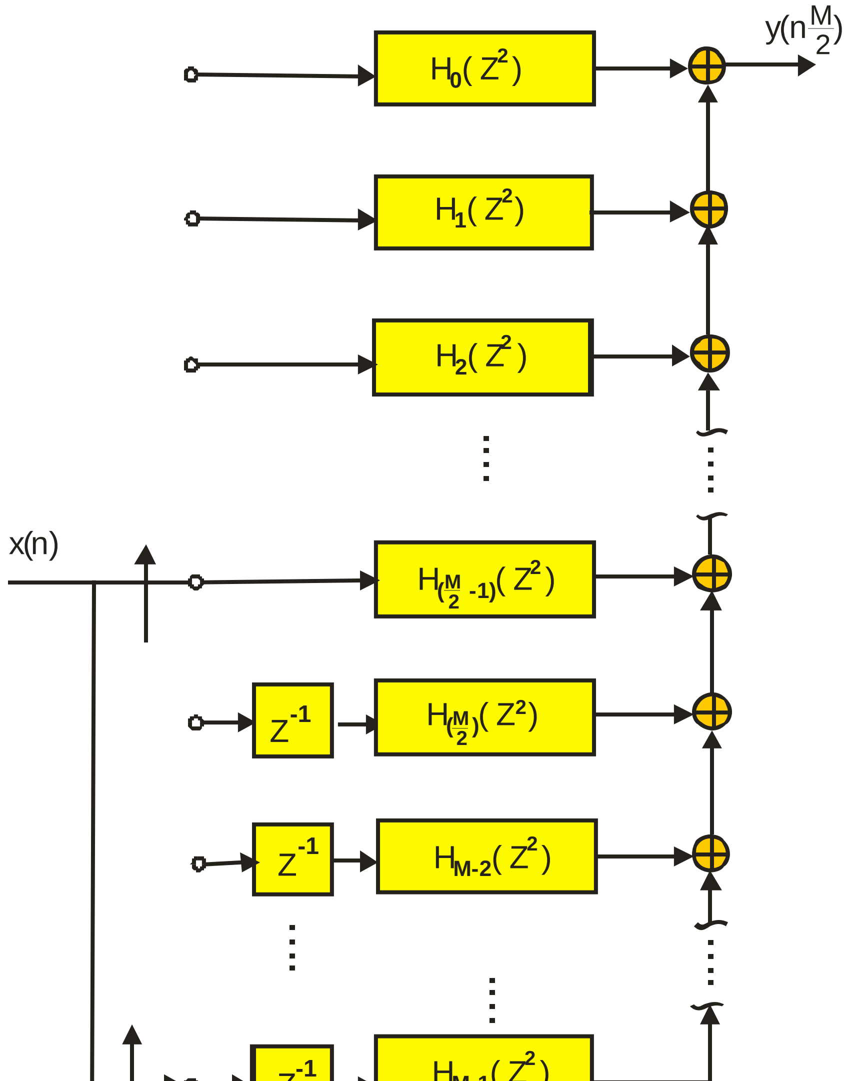
Path Filters:
Polynomials in
Z^M
Converted to
Polynomials in
Z^2



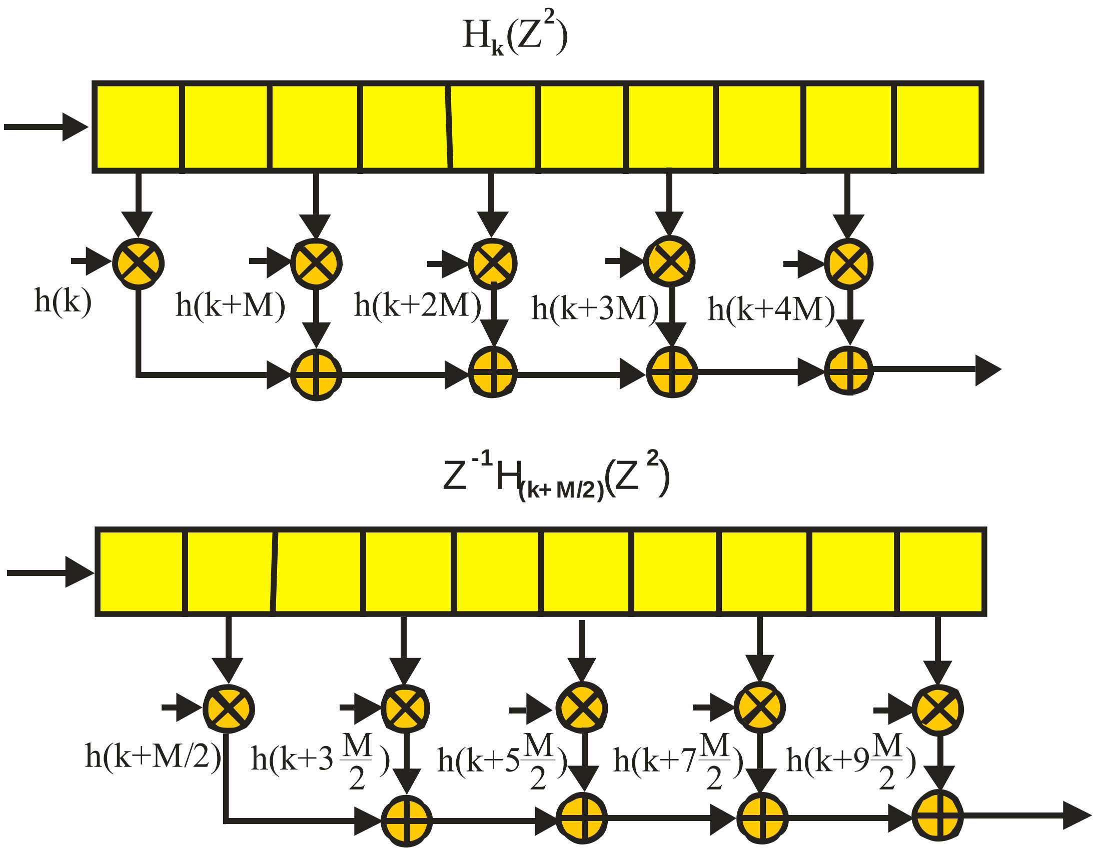
Use Noble Identity to Pull M/2-to-1 Resampler Through Delays in Lower Half of Paths



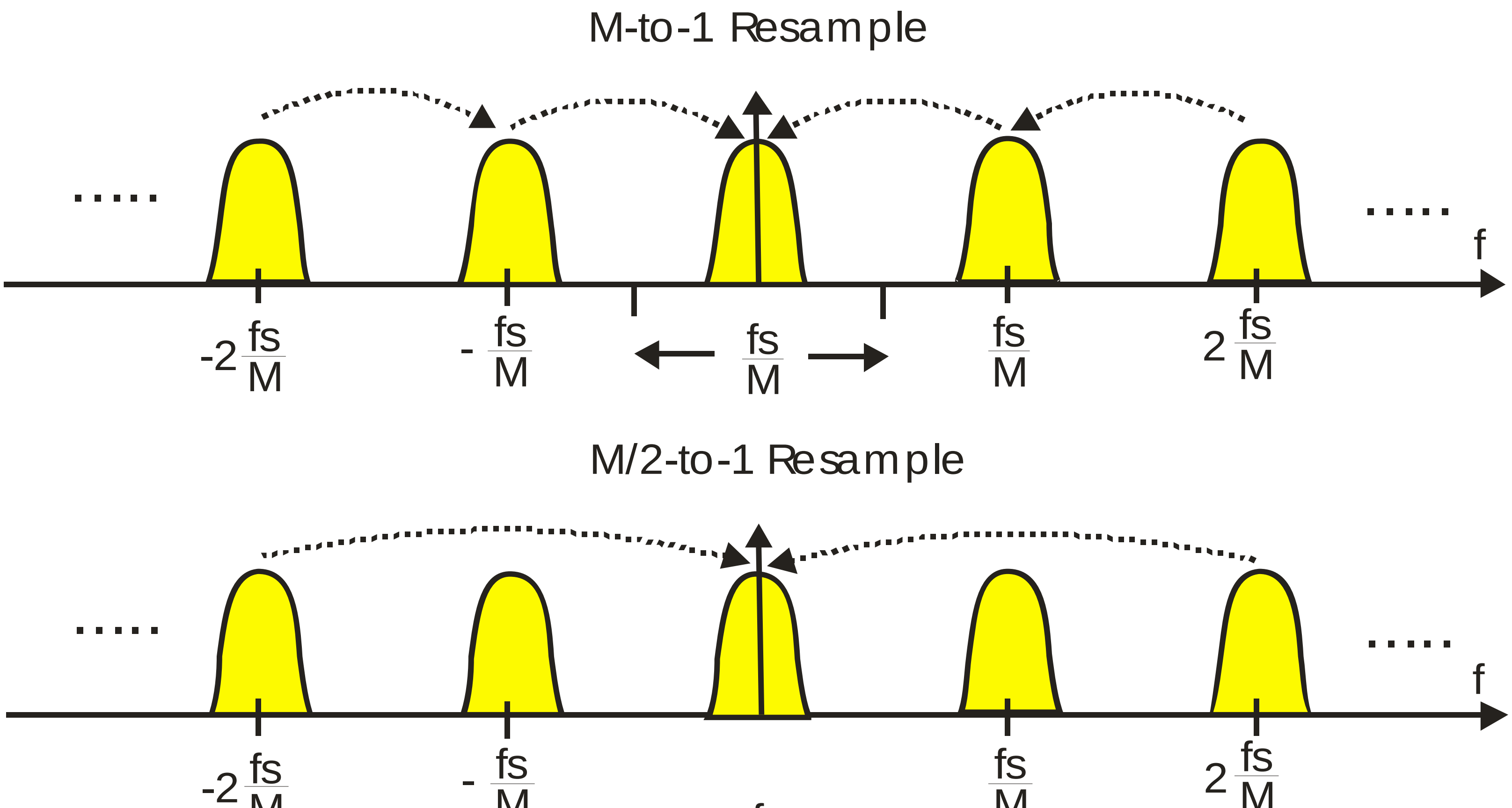
Replace Delays and M/2-to-1 Resamplers with Dual Input M/2 Path Commutator



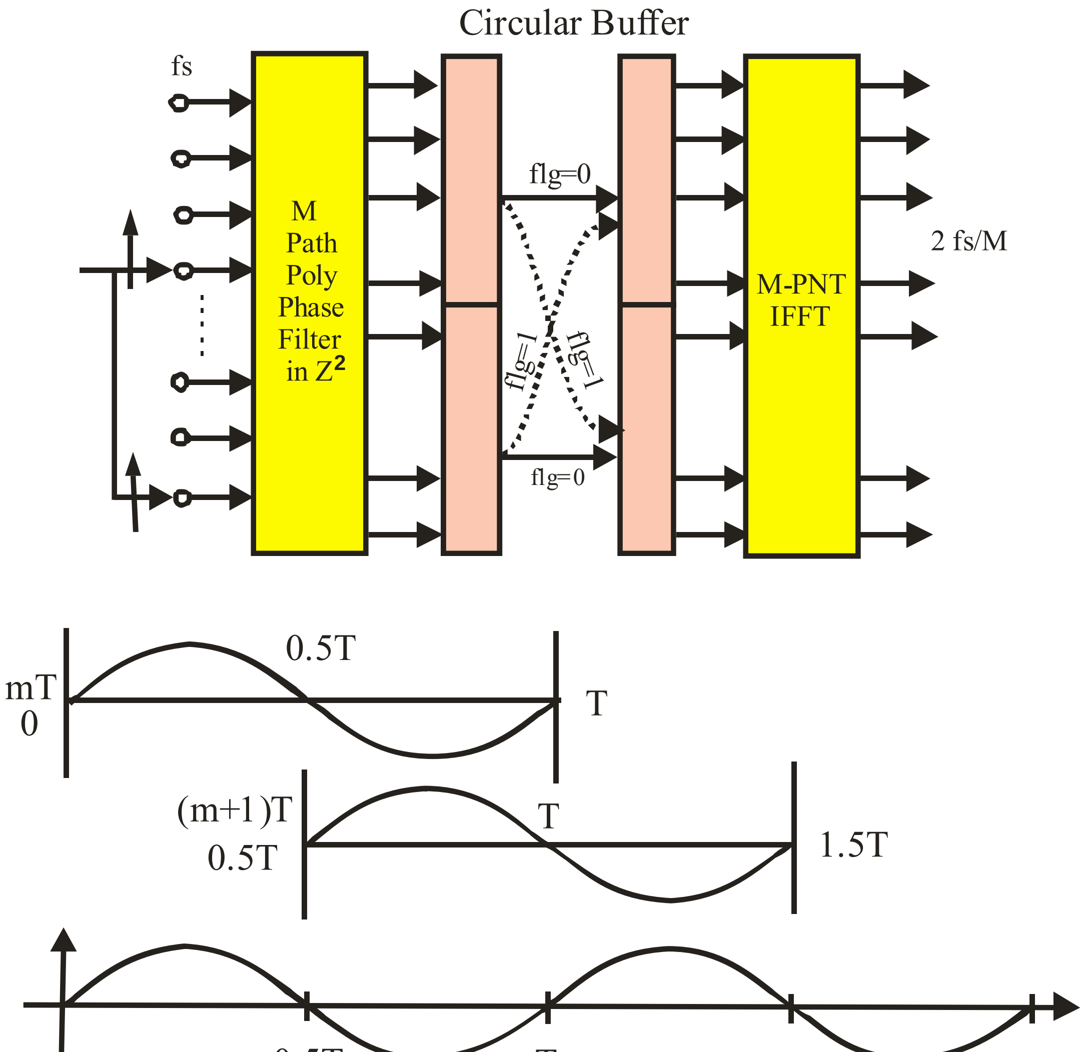
Fold Unit Delays in lower half Filter Paths Into Filter Polynomials in Z^2



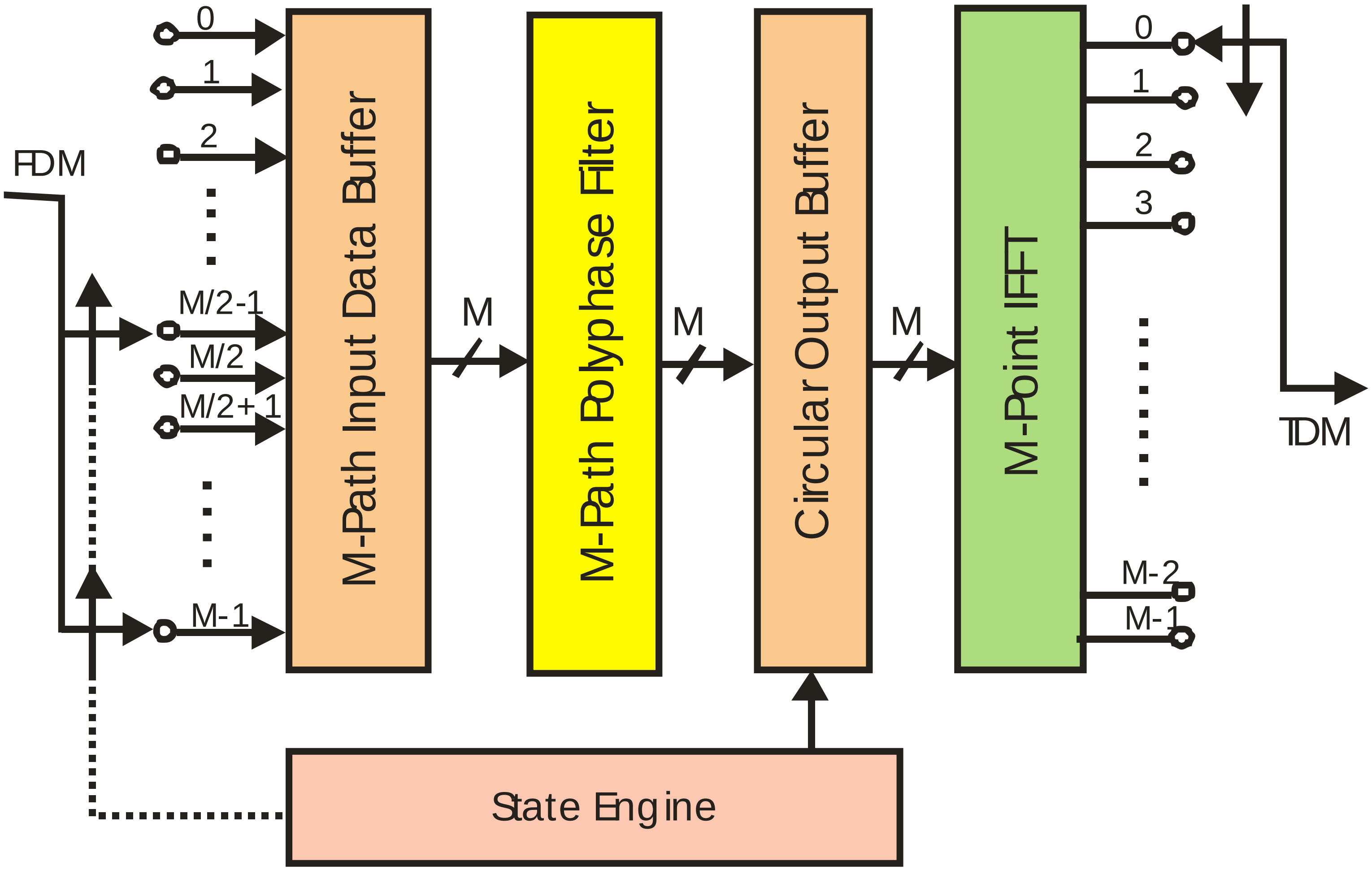
M-to-1 Down Sample Aliases Multiples of Output Sample Rate to DC
M/2-to-1 Down Sample Aliases Odd Multiples of Output Sample Rate to Half sample Rate



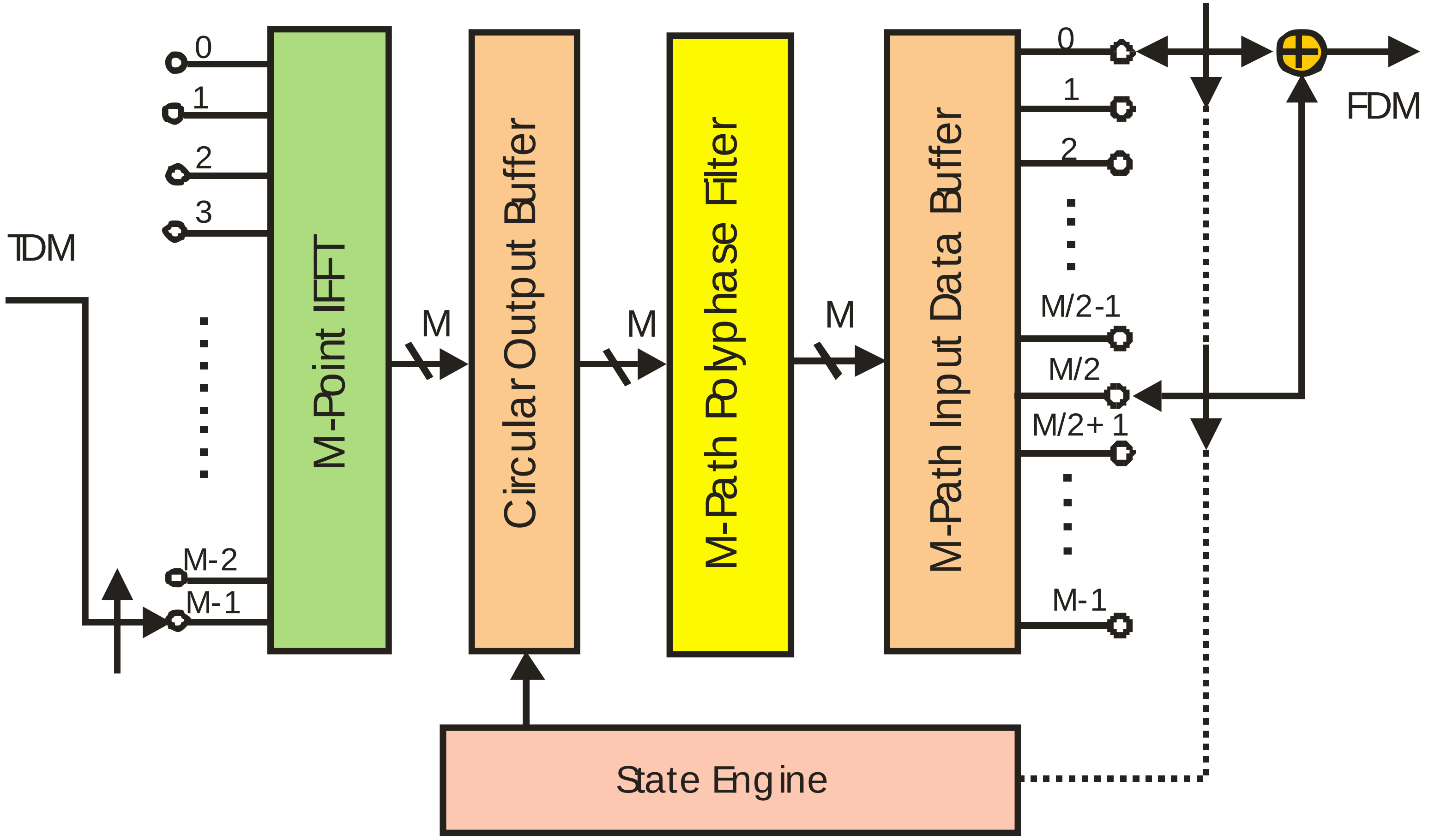
Circular Buffer Between Polyphase Filter and IFFT Aligns Shifting Input Origin with IFFT's Origin



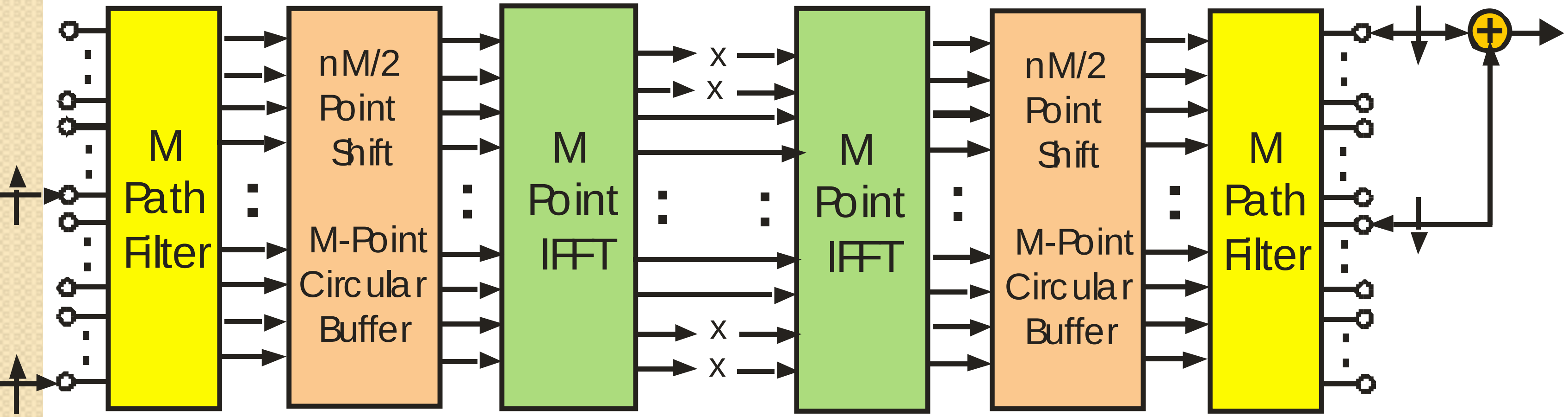
M/2-to-1 Analysis Channelizer



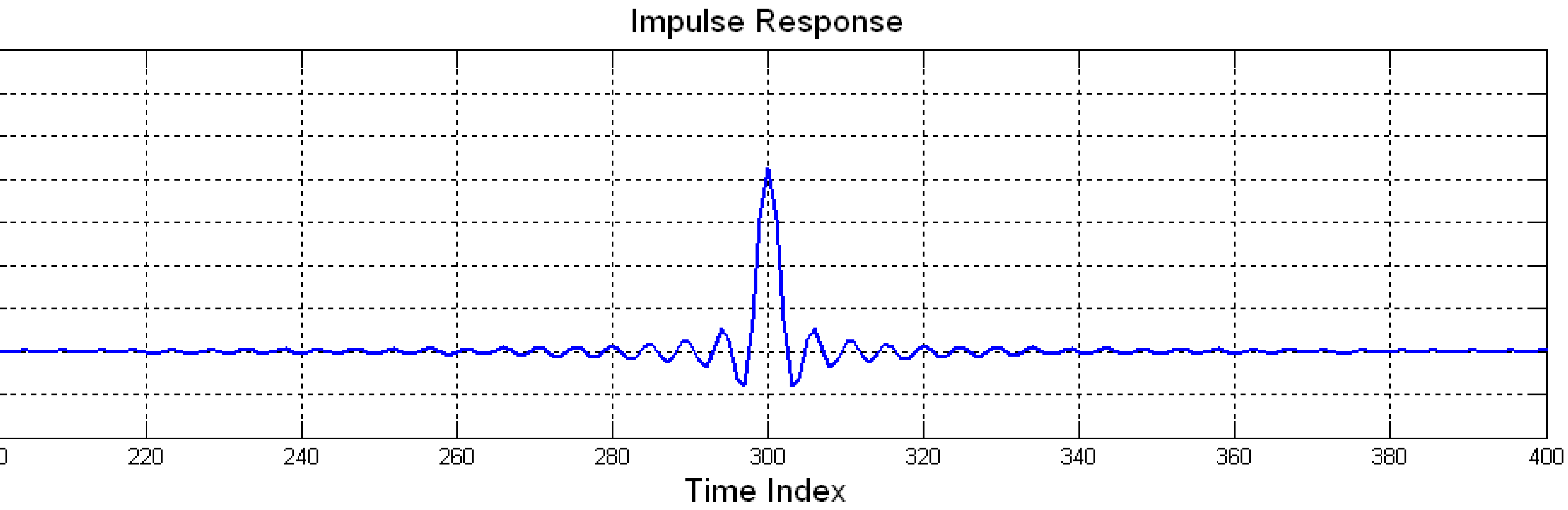
1-to-M/2 Synthesis Channelizer

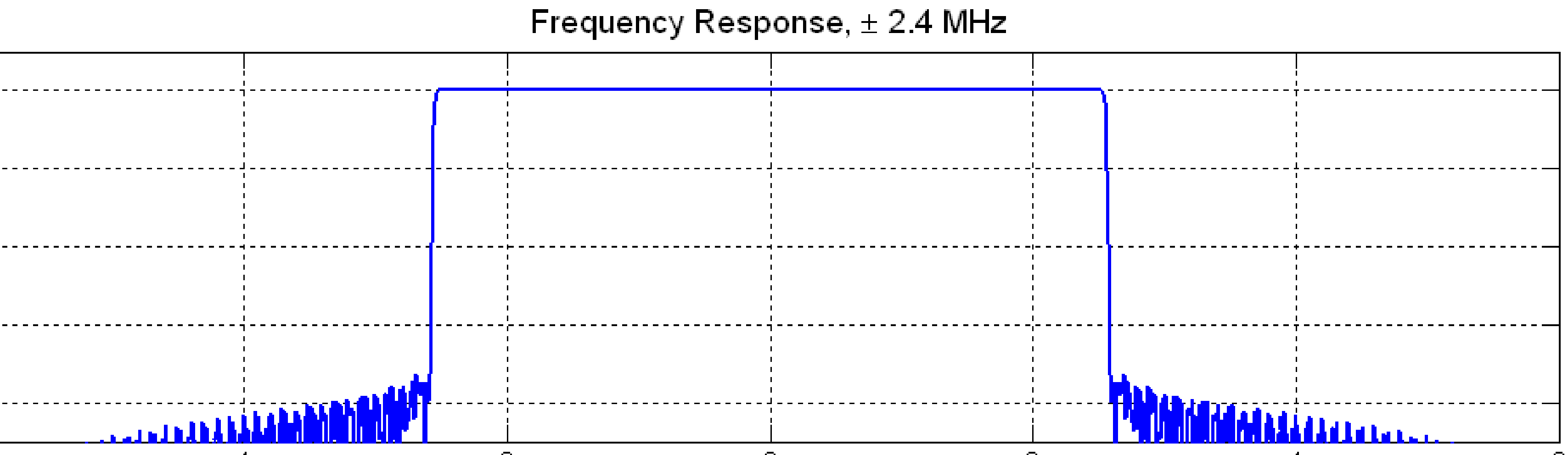


Frequency Domain Filtering With Cascade M/2-to-1 Analysis and 1-to-M/2 Synthesis Channelizers

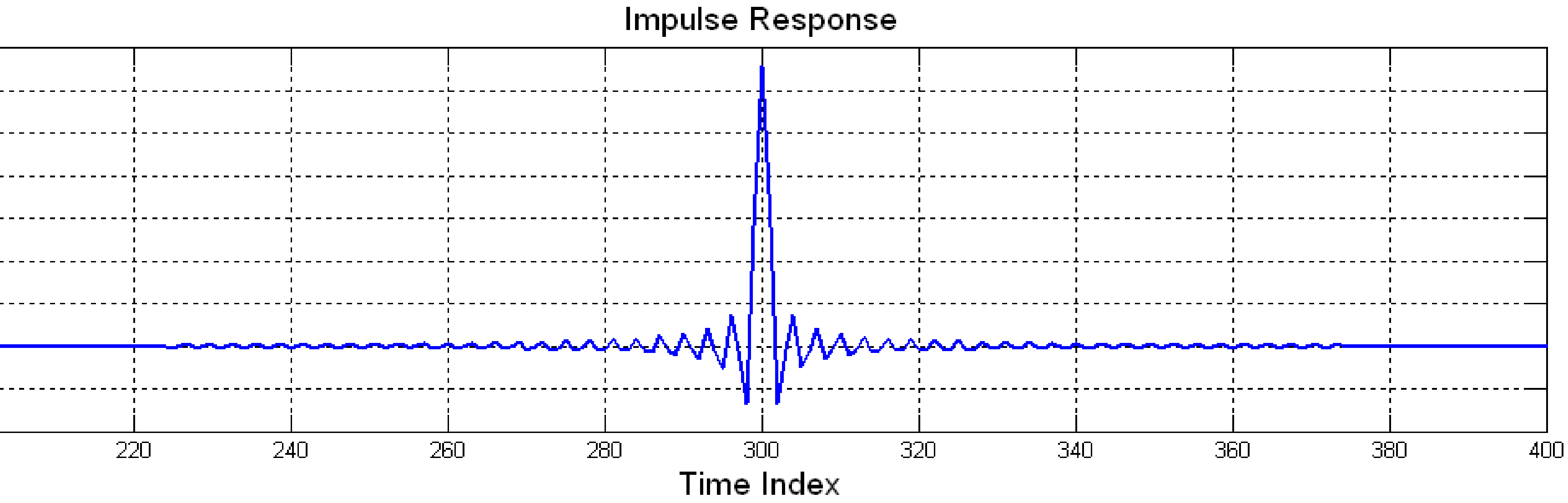


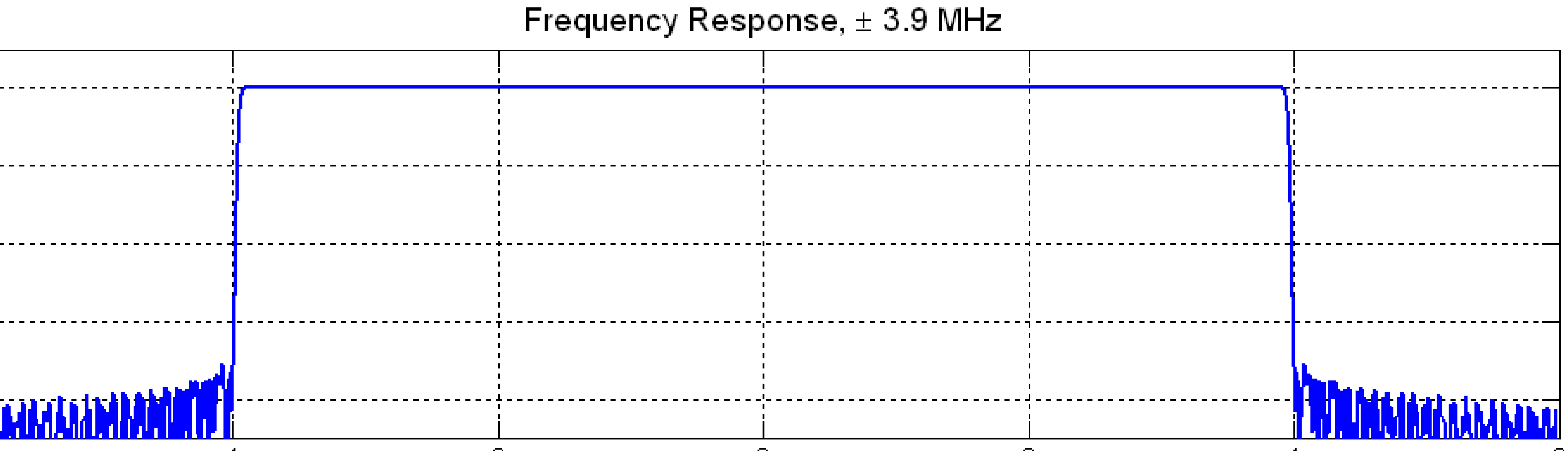
Impulse response and Frequency Response
25-Enabled Ports: 2.4 MHz Bandwidth


Impulse Response


Frequency Response, ± 2.4 MHz

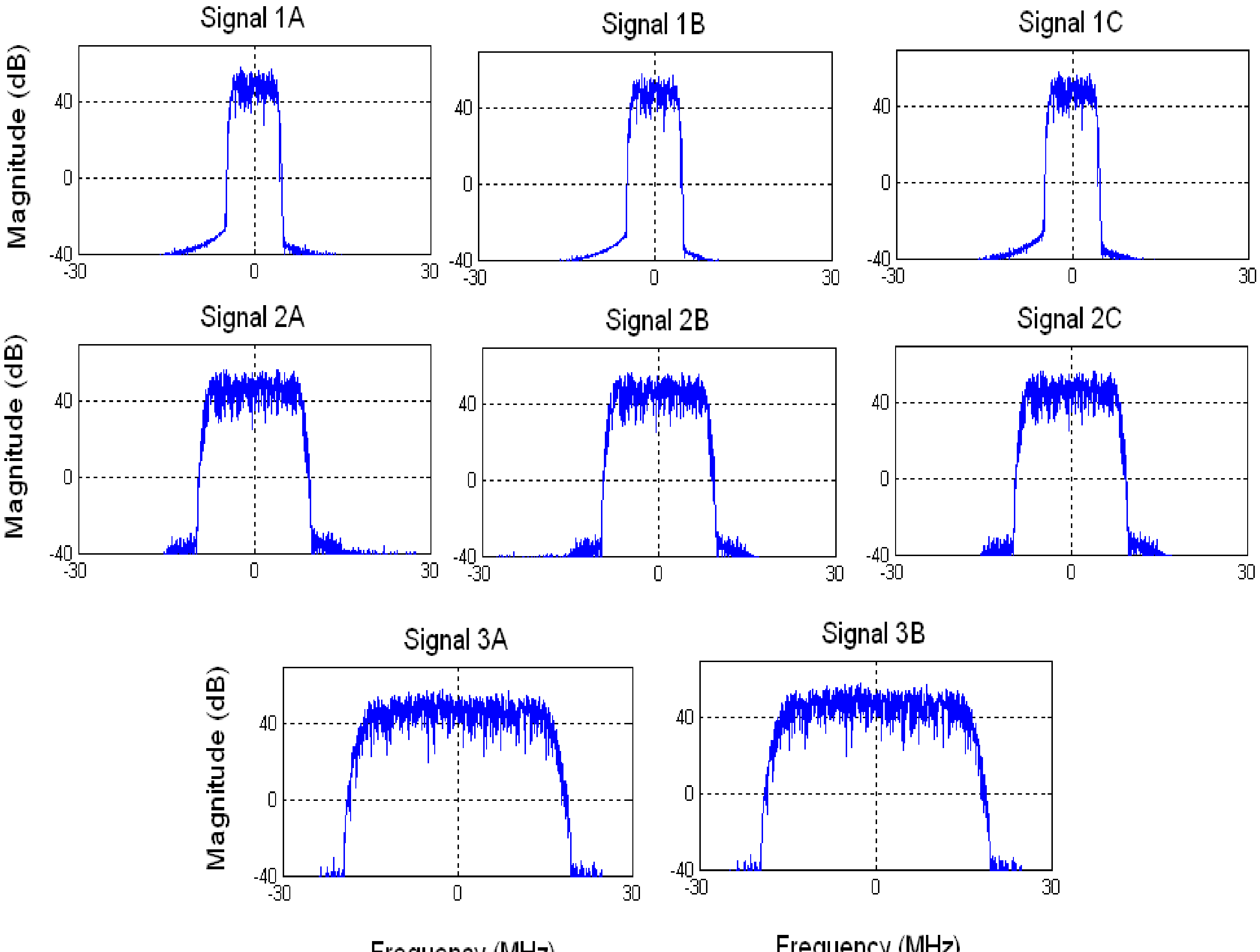
Impulse Response and Frequency Response
40-Enabled Ports: 3.9 MHz Bandwidth



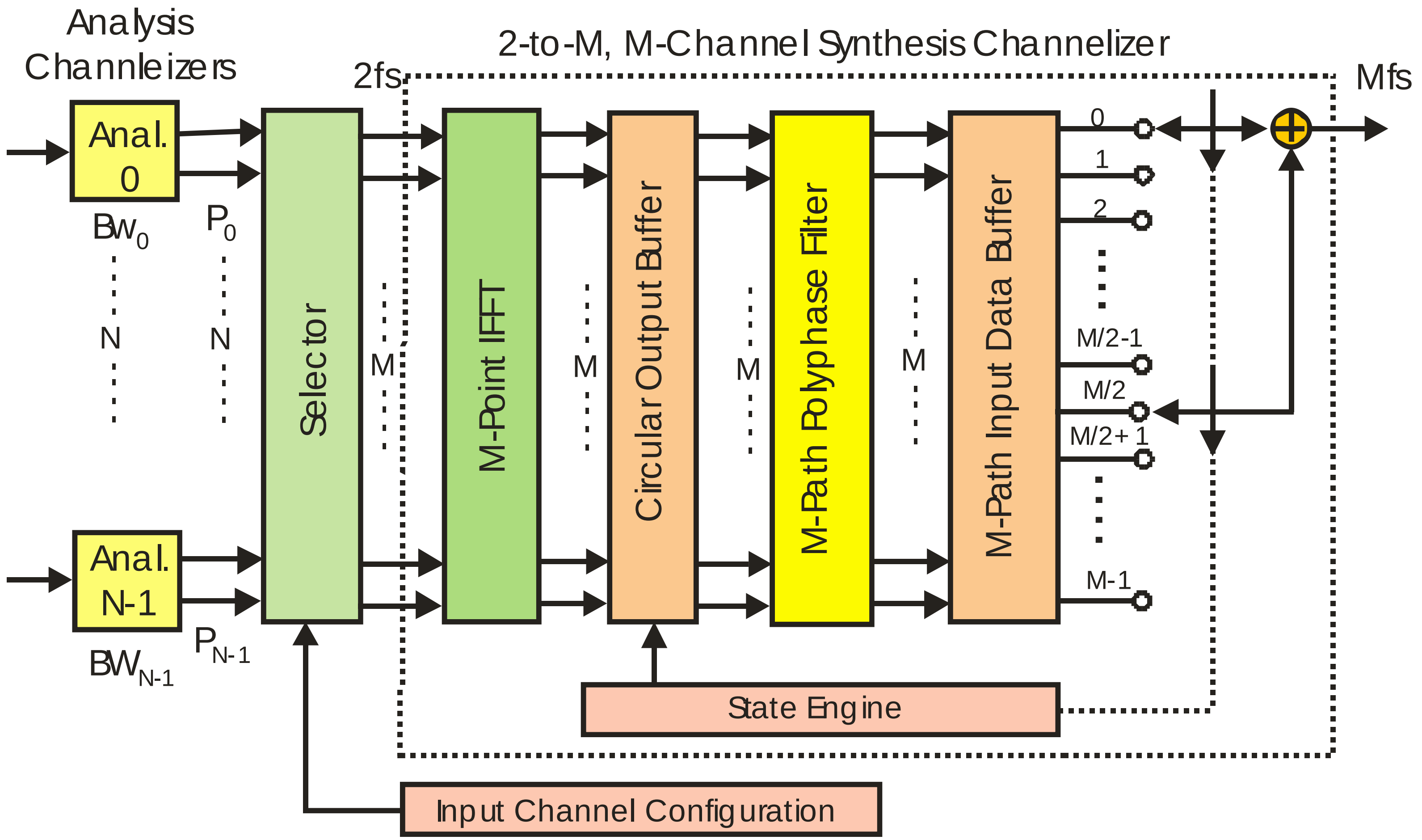


Mixed, Arbitrary Bandwidth Channelizers

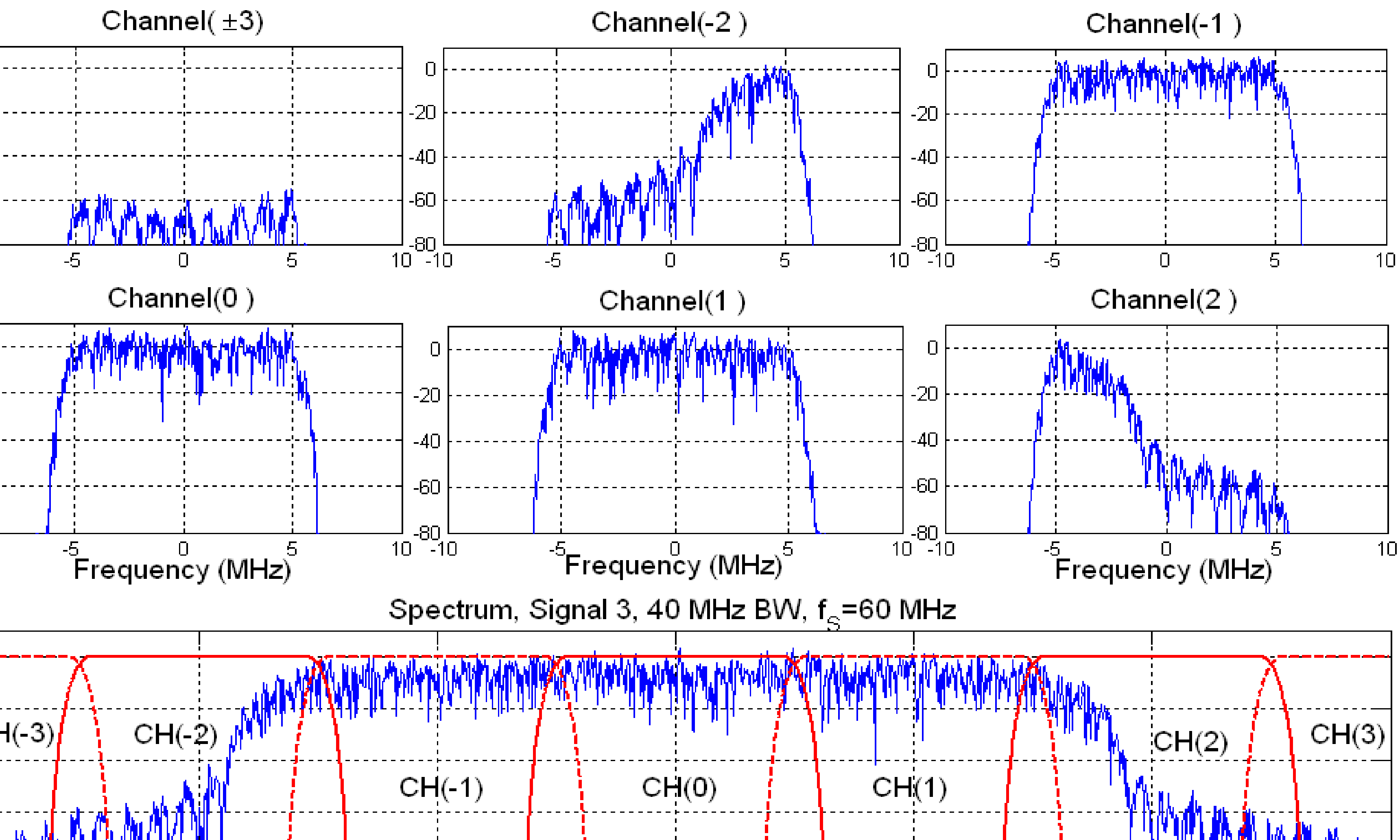
Mixed Bandwidth Signals presented to Channel Synthesizer



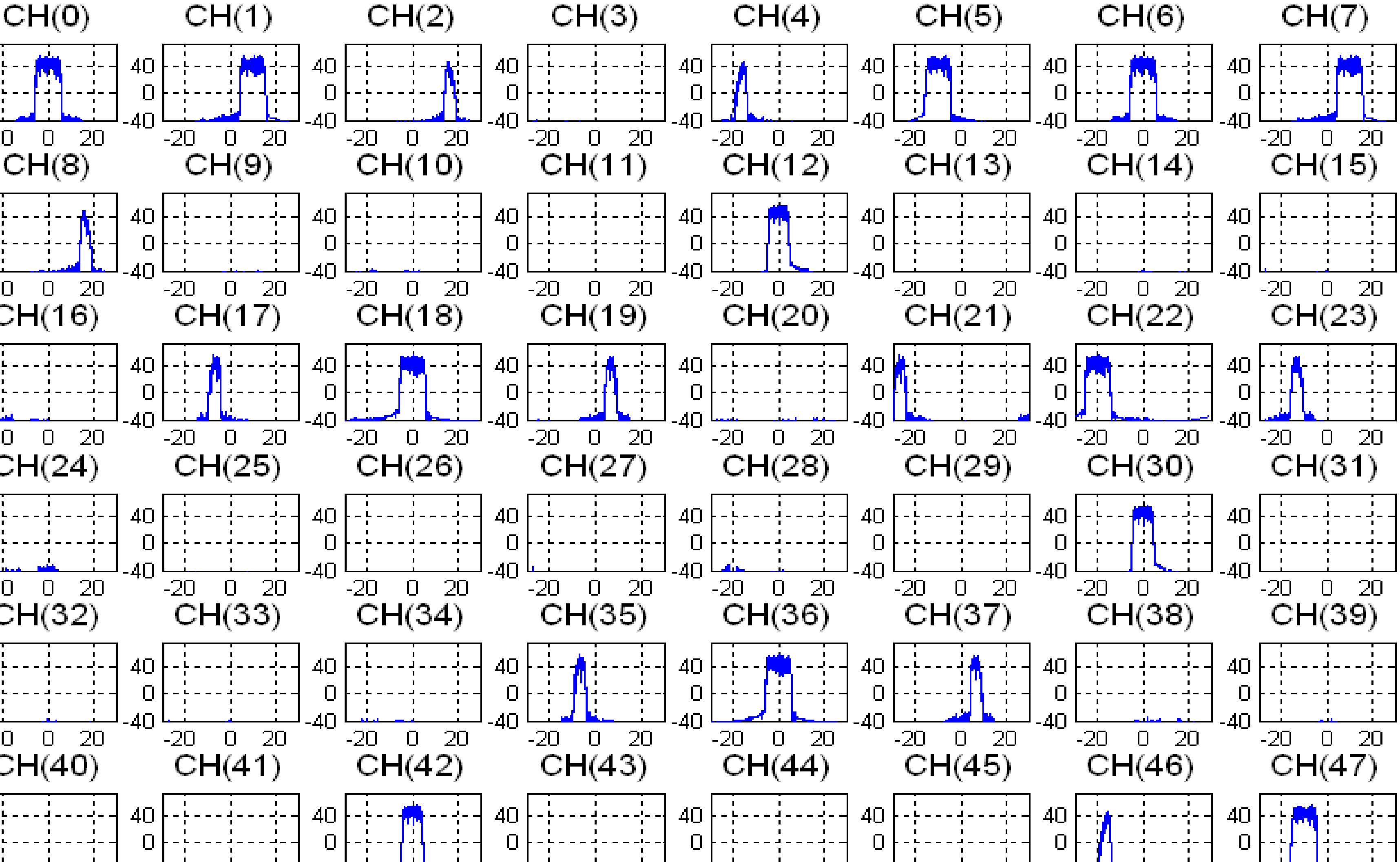
ompose Broadband Signals Using Short Analysis rs and Present Components to Synthesizer



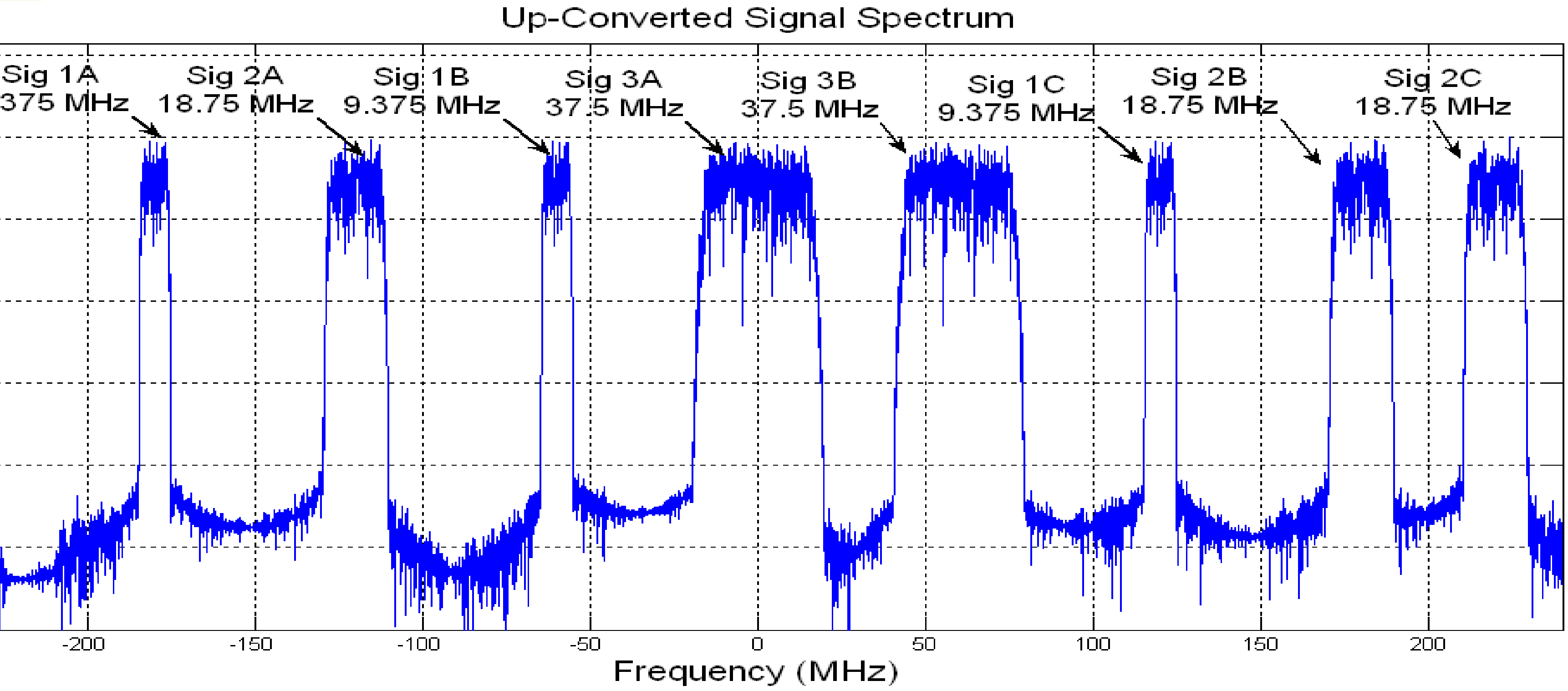
0-MHz Input Signal Partitioned into ive 10-MHz Sub Channels: f_S=20 MHz



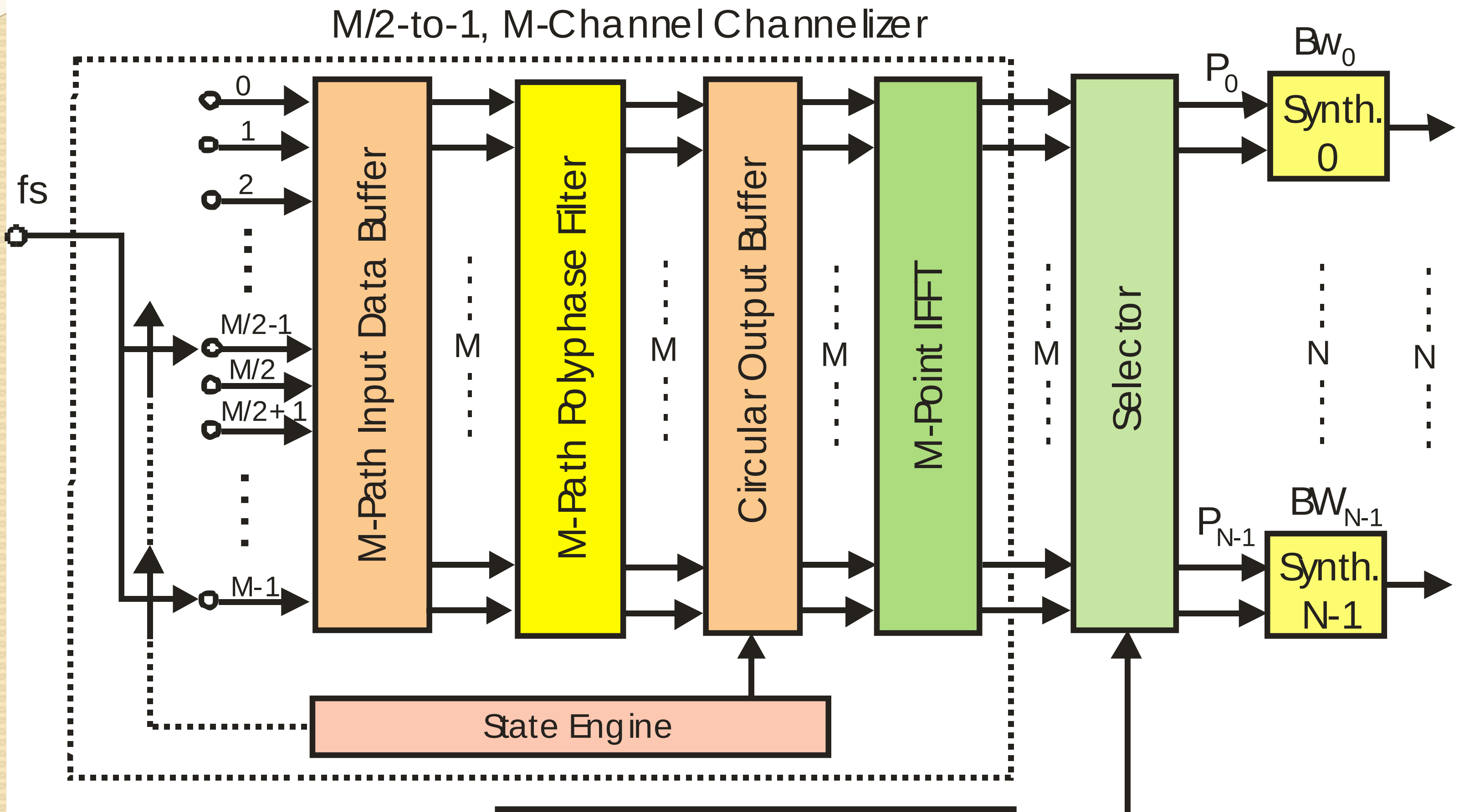
ultiple Partitioned Input Bands resented to Synthesizer



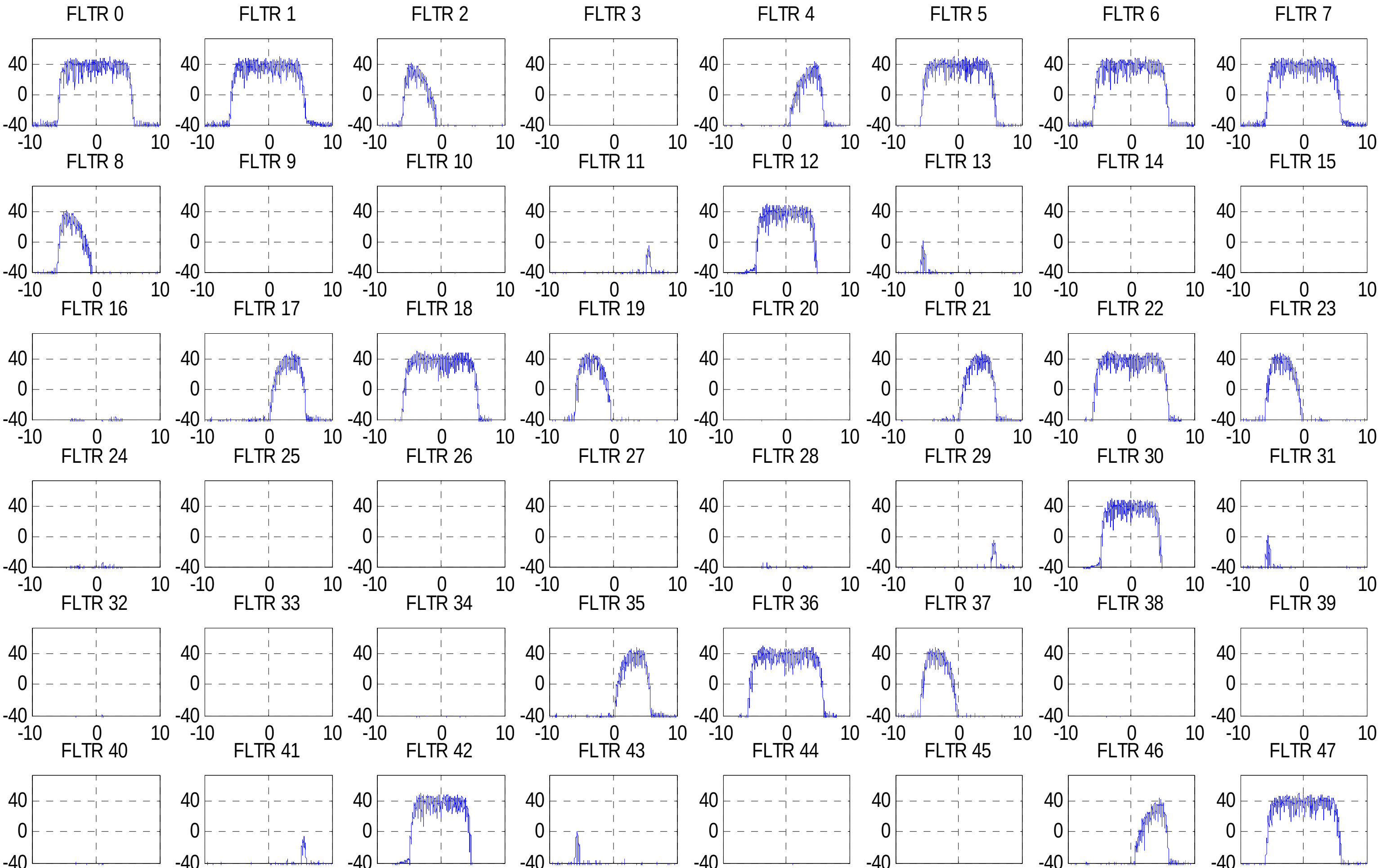
Assembled Multiple BW Channels in Single Synthesis Channelizer



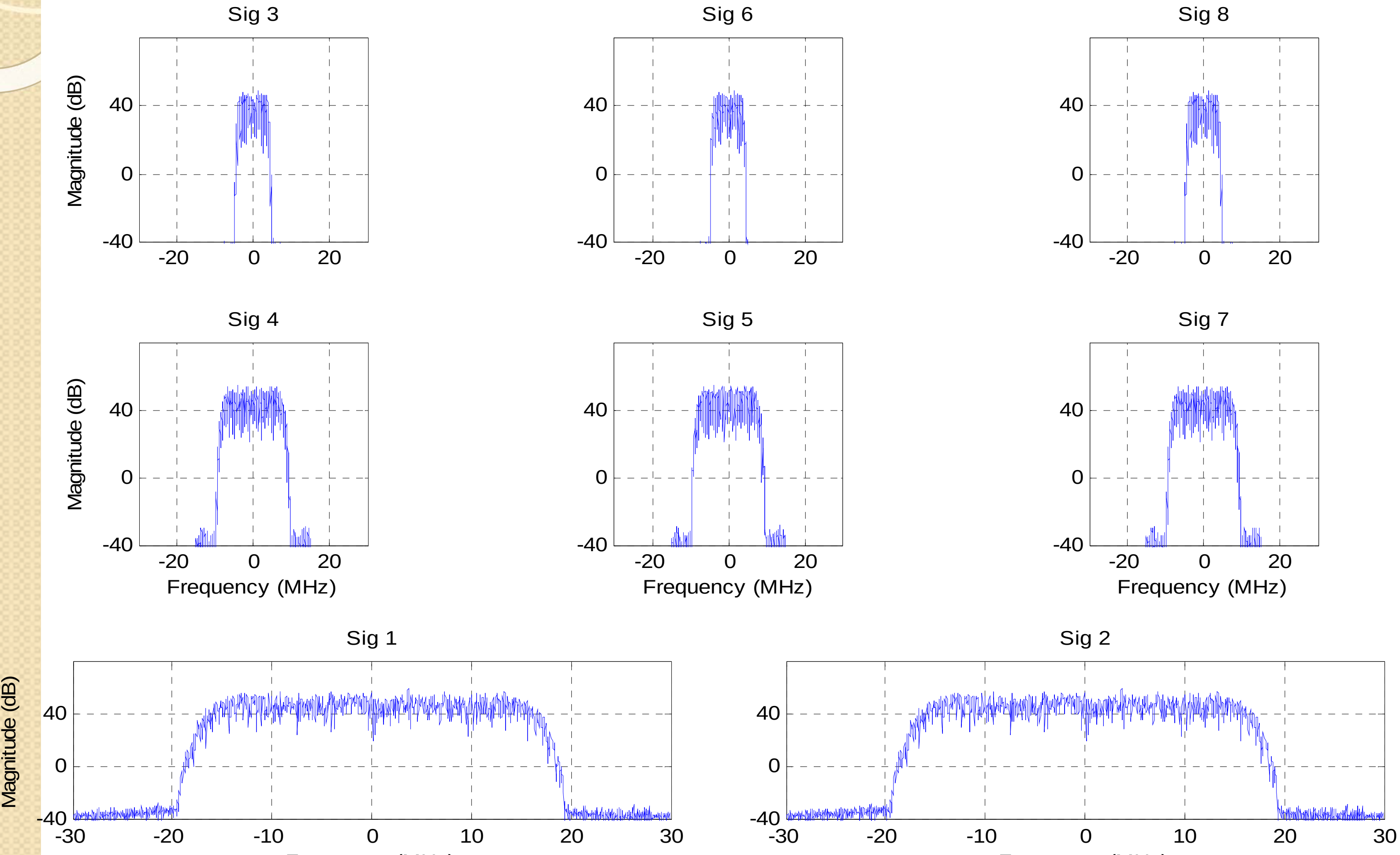
Reassemble Decomposed Broadband Signals Using Short Synthesis Filters formed by Multiple Channel Analysis Channelizer



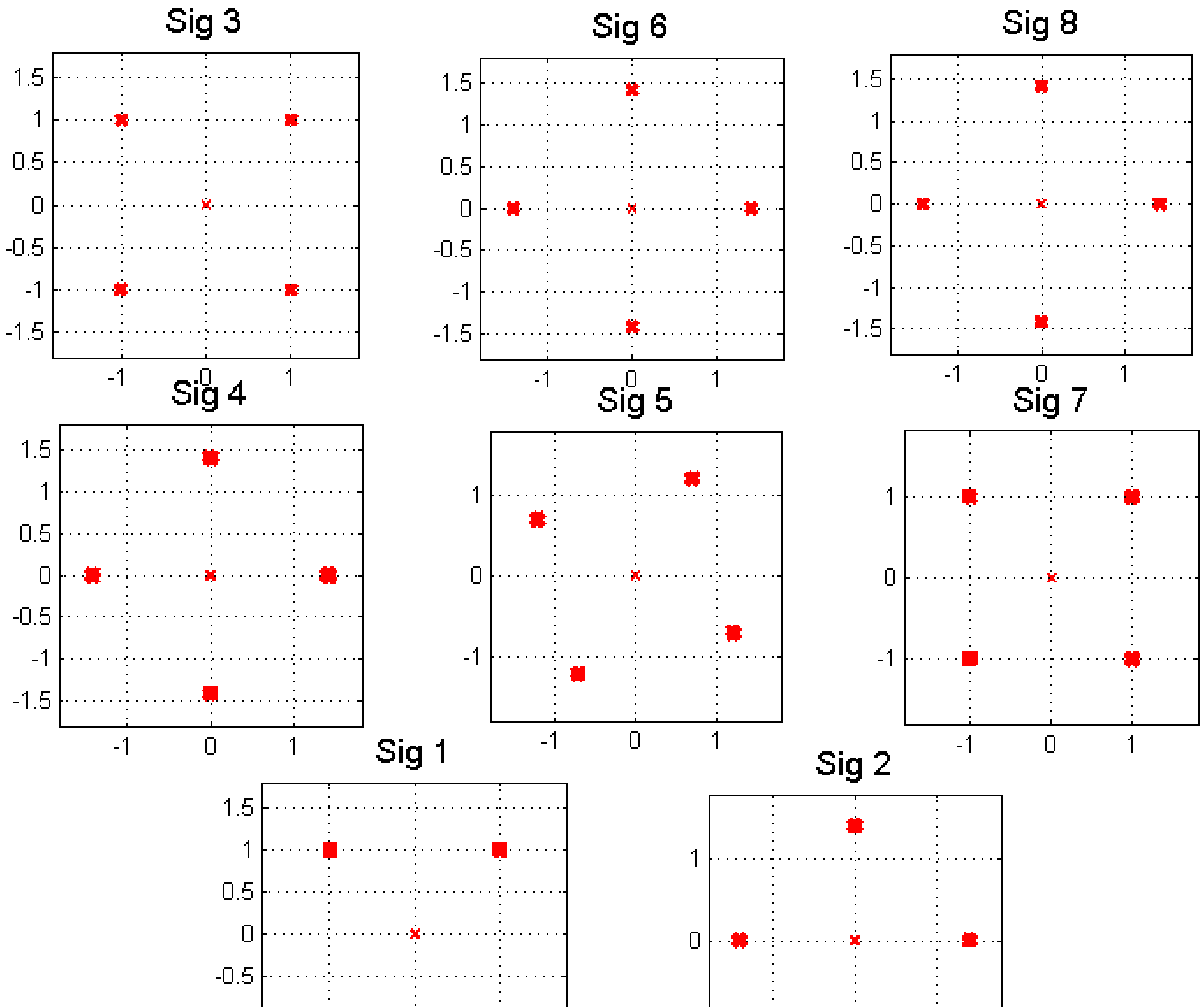
Partitioned spectral Components from ingle Multi-Channel Analyzer



Reassembled Wide band Channels rom Short Synthesis Channelizers

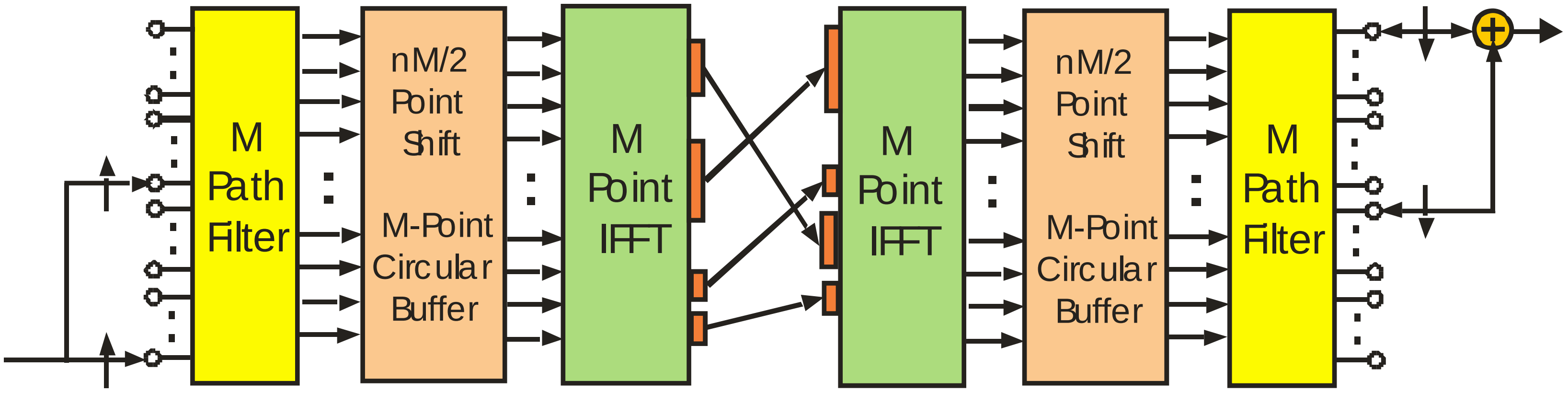


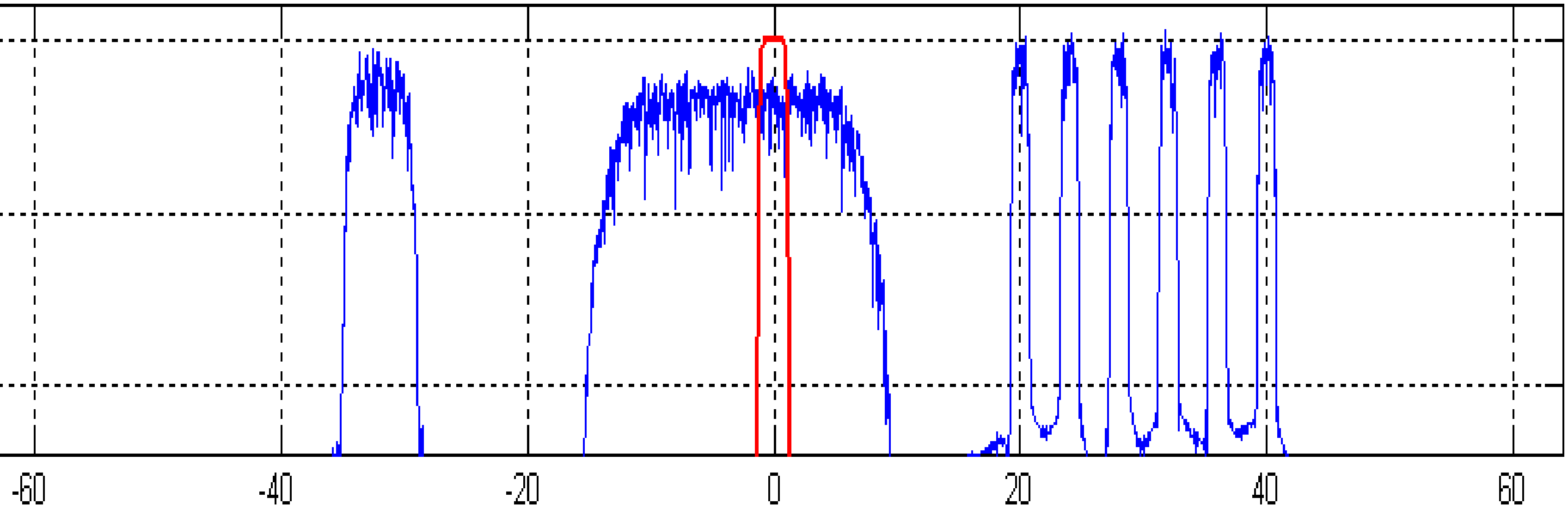
Signal Fidelity Preserved under Multiple Sub-Channel Disassembly and Reassembly



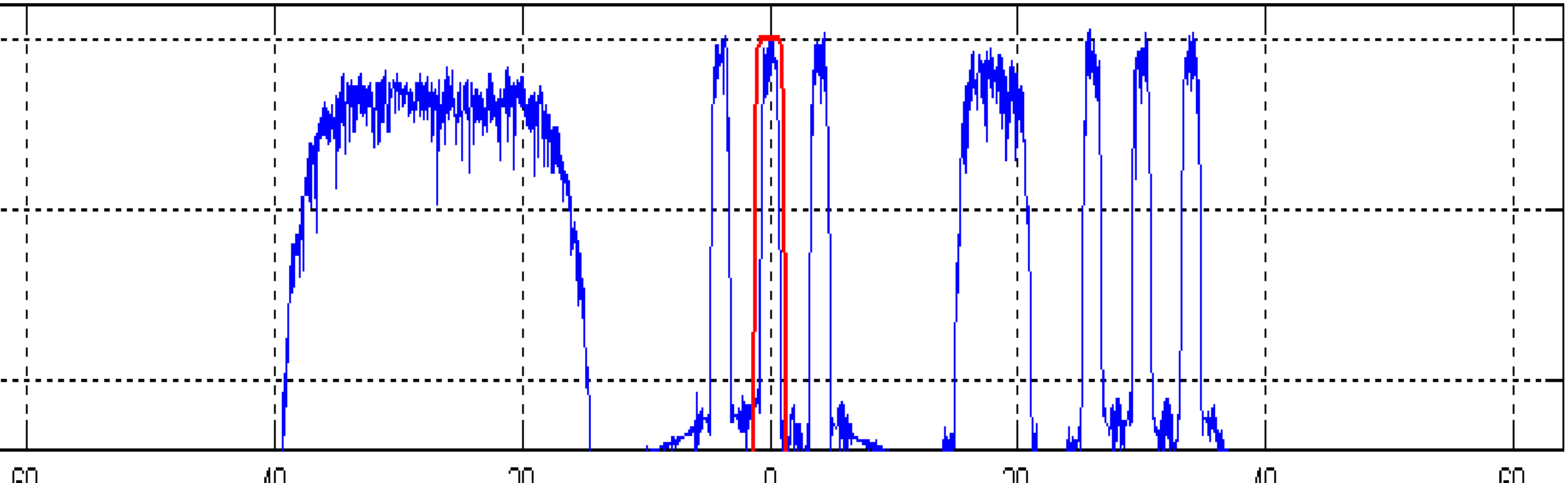

AZAM!
BLAM!
AS BILLY SPEAKS THE MAGIC WORD HE BECOMES CAPTAIN
CAPTAIN MARVEL, I SALUTE YOU. HENCEFORTH IT SHALL BE YOUR SACRED DUTY TO DEFEND THE POOR AND HELPLESS, RIGHT WRONGS AND CRUSH EVIL EVERYWHERE.
YES, SIRE.

Cascade M/2-to-1 Analysis and
1-to-M/2 Synthesis Channelizers
Frequency Domain Filtering and Spectral Shuffle

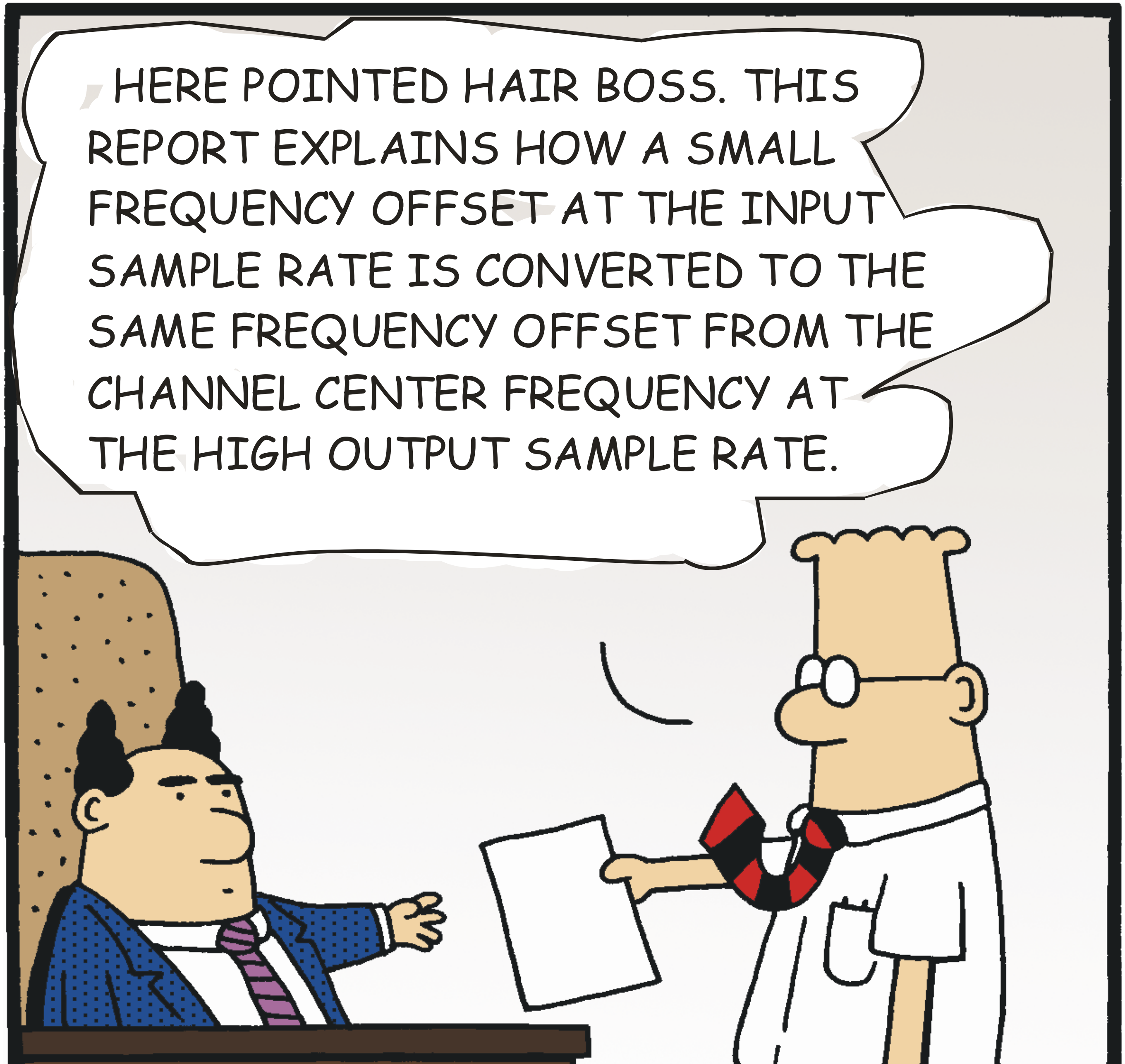




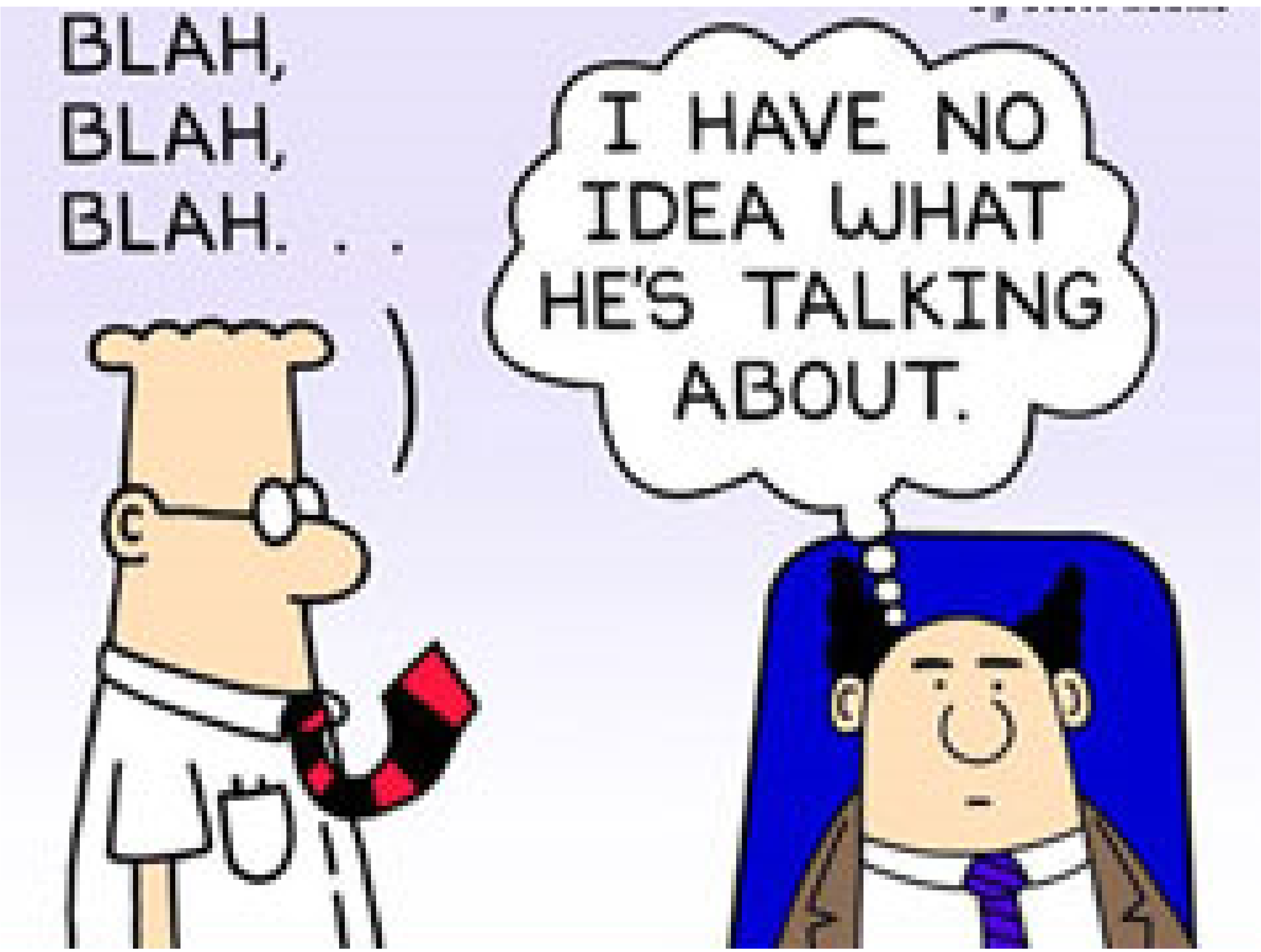
Composite Multi-Channel Input Spectrum

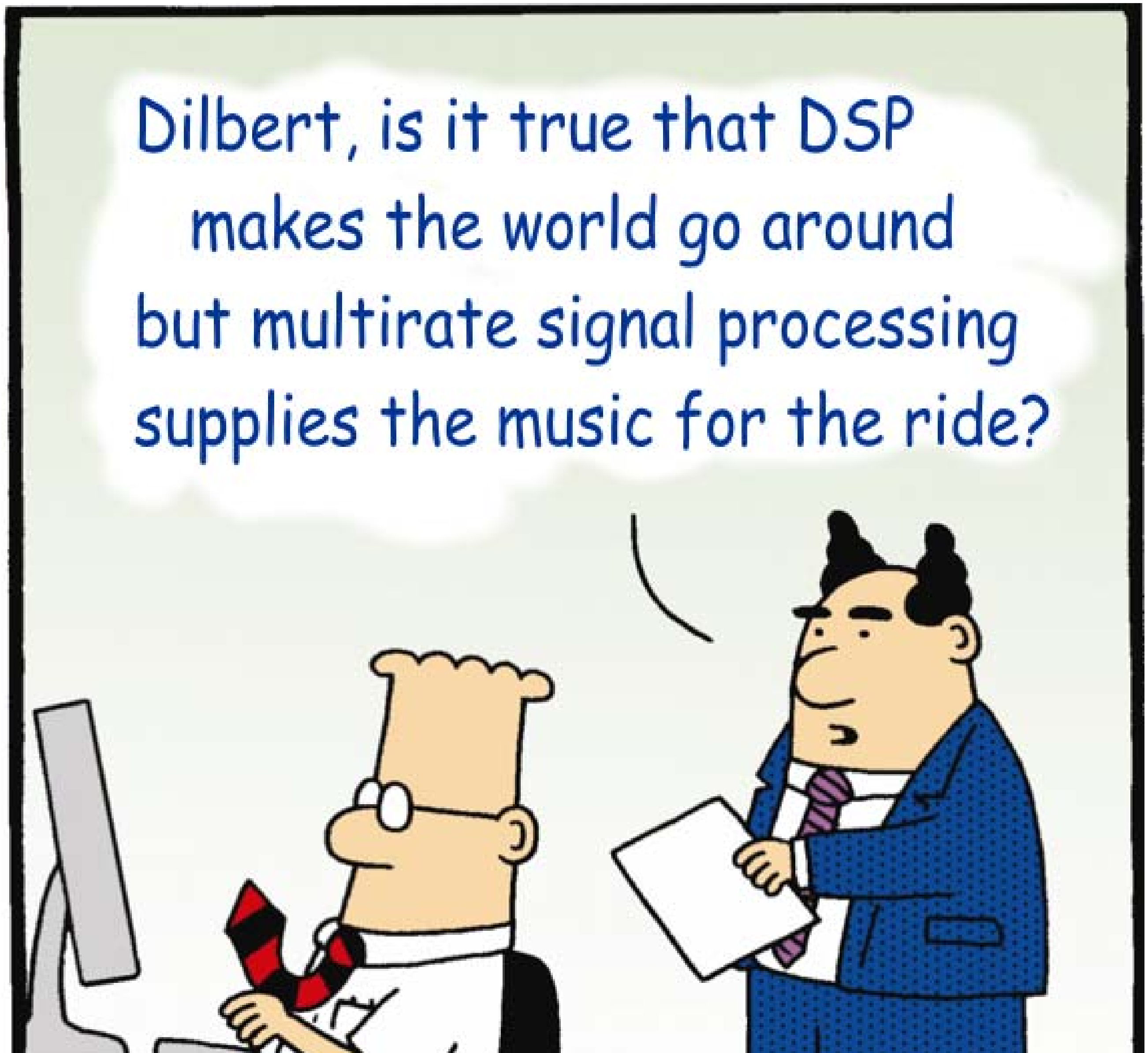


Composite Analysis, Rearranged, and Synthesized Output Spectrum


HERE POINTED HAIR BOSS. THIS REPORT EXPLAINS HOW A SMALL FREQUENCY OFFSET AT THE INPUT SAMPLE RATE IS CONVERTED TO THE SAME FREQUENCY OFFSET FROM THE CHANNEL CENTER FREQUENCY AT THE HIGH OUTPUT SAMPLE RATE.

Suspicions Confirmed!




Dilbert, is it true that DSP makes the world go around but multirate signal processing supplies the music for the ride?

Is There any Doubt???